DUST-GAS TEMPERATURE DISEQUILIBRIUM AT HIGH ALTITUDES IN THE MARS ATMOSPHERE

by

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ABSTRACT

A model to predict dust temperatures in the Mars atmosphere is presented to demonstrate under which conditions dust grains are not in equilibrium with the atmospheric gas. Previous models have assumed an equilibrium in which dust and gas temperature are the same. Here a radiative-conductive balance is preformed. A range of particle sizes are treated separately. Scattering as predicted by Mie theory is used for the solar-band heating term, but not the infrared heating term, where scattering is negligible. Dust grains are taken to cool as blackbodies and lose energy to atmospheric gas through collisions, proportional to gas density. Dust grains are found to be in equilibrium with the atmospheric gas at the lowest altitudes, where gas pressure is greatest. At a predictable pressure dust temperature begins to decouple from gas temperature, as dust grain temperature is increasingly dominated by radiative absorption and emission instead of collisions with gas molecules.
for my parents, who made this possible,
and Nate, who made it enjoyable
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# TABLE OF CONTENTS

| LIST OF TABLES                        | viii |
| LIST OF FIGURES                      | ix  |
| LIST OF SYMBOLS                      | xi  |

## CHAPTER

1. INTRODUCTION .......................................................... 1

1.1. Why study Mars dust? .............................................. 1

1.2. Dust in Martian weather ........................................... 5

2. BACKGROUND .................................................................. 7

2.1. Basic concepts in radiation transfer ............................ 7

2.2. Scattering .................................................................. 10

2.2.1. Single particle Mie Theory ...................................... 11

2.2.2. Mie scattering over multiple spheres ......................... 12

2.3. Numerical Approaches to Radiation Transfer .................... 13

2.4. Past Work .................................................................. 16

2.4.1. Overview: Radiative Transfer Studies ......................... 16

2.4.2. Deriving Indices of Refraction ................................. 18

2.4.3. Size Distribution ................................................. 20

2.4.4. Mixing Ratio ..................................................... 20

2.4.5. Global Climatology ............................................... 23

2.4.6. Dust in GCMs ..................................................... 24

2.4.7. Heating Rates ..................................................... 24
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 Objective of This Study</td>
<td>25</td>
</tr>
<tr>
<td>3. METHOD</td>
<td>26</td>
</tr>
<tr>
<td>3.1. Atmospheric Temperature Profiles</td>
<td>26</td>
</tr>
<tr>
<td>3.2. Dust Property Inputs</td>
<td>28</td>
</tr>
<tr>
<td>3.3. Radiative Transfer</td>
<td>35</td>
</tr>
<tr>
<td>3.4. Radiative-conductive Equilibrium</td>
<td>36</td>
</tr>
<tr>
<td>4. RESULTS AND ANALYSIS</td>
<td>40</td>
</tr>
<tr>
<td>4.1. The Base Model</td>
<td>40</td>
</tr>
<tr>
<td>4.2. “Clear” vs. Dusty Atmospheres</td>
<td>42</td>
</tr>
<tr>
<td>4.3. Varying Temperature Profile</td>
<td>44</td>
</tr>
<tr>
<td>4.4. Parametric Uncertainty Analysis</td>
<td>52</td>
</tr>
<tr>
<td>4.4.1. Indices of Refraction</td>
<td>52</td>
</tr>
<tr>
<td>4.4.2. Ground Temperature, Optical Depth, and Reflectivity</td>
<td>57</td>
</tr>
<tr>
<td>4.4.3. Dust Distribution</td>
<td>60</td>
</tr>
<tr>
<td>4.5. Other sources of uncertainty</td>
<td>64</td>
</tr>
<tr>
<td>4.6. Effect of dust temperature on infrared spectra</td>
<td>67</td>
</tr>
<tr>
<td>4.7. Effect of dust temperature on heating rates</td>
<td>68</td>
</tr>
<tr>
<td>5. DISCUSSION</td>
<td>71</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>78</td>
</tr>
<tr>
<td>APPENDIX</td>
<td></td>
</tr>
<tr>
<td>A MIE THEORY NUMERICAL SOLUTION</td>
<td>86</td>
</tr>
<tr>
<td>B INTERACTIVE DATA LANGUAGE PROGRAMS</td>
<td>90</td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>3-1 Parameters that go into the present model, and the output expected.</td>
<td>29</td>
</tr>
<tr>
<td>3-2 Base model values used for the variables in the vertical particle distribution.</td>
<td>32</td>
</tr>
<tr>
<td>3-3 Base model, clear atmosphere, input values.</td>
<td>33</td>
</tr>
<tr>
<td>3-4 Parameters for test cases. Based on Toon et al. [1989], Table 4.</td>
<td>36</td>
</tr>
<tr>
<td>4-1 Pressure of gas-dust temperature decoupling.</td>
<td>42</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1</td>
<td>Clear and dusty temperature profiles</td>
<td>27</td>
</tr>
<tr>
<td>3-2</td>
<td>Temperature profiles derived from lander descent measurements</td>
<td>28</td>
</tr>
<tr>
<td>3-3</td>
<td>Size distribution at the lowest atmospheric layer for the base model.</td>
<td>31</td>
</tr>
<tr>
<td>3-4</td>
<td>Number density vs. altitude.</td>
<td>32</td>
</tr>
<tr>
<td>4-1</td>
<td>Particle temperature with altitude</td>
<td>41</td>
</tr>
<tr>
<td>4-2</td>
<td>Clear vs. dusty case particle temperatures</td>
<td>43</td>
</tr>
<tr>
<td>4-3</td>
<td>Dust-gas temperature difference</td>
<td>44</td>
</tr>
<tr>
<td>4-4</td>
<td>Dust temperatures with Viking I temperature profile</td>
<td>45</td>
</tr>
<tr>
<td>4-5</td>
<td>Dust temperatures with Viking II temperature profile</td>
<td>46</td>
</tr>
<tr>
<td>4-6</td>
<td>Dust temperature in isolation from gas</td>
<td>47</td>
</tr>
<tr>
<td>4-7</td>
<td>Absorption efficiency vs. radius in the visible and infrared</td>
<td>48</td>
</tr>
<tr>
<td>4-8</td>
<td>Maximum dust temperature vs. radius</td>
<td>49</td>
</tr>
<tr>
<td>4-9</td>
<td>Case of dust at blackbody temperature</td>
<td>50</td>
</tr>
<tr>
<td>4-10</td>
<td>Dust temperatures with Pathfinder temperature profile</td>
<td>51</td>
</tr>
<tr>
<td>4-11</td>
<td>Dust-gas temperature differences for lander temperature profiles</td>
<td>51</td>
</tr>
<tr>
<td>4-12</td>
<td>Sensitivity of dust temperature to indices of refraction</td>
<td>53</td>
</tr>
<tr>
<td>4-13</td>
<td>Dust temperature vs. radius, varying indices of refraction</td>
<td>53</td>
</tr>
<tr>
<td>4-14</td>
<td>Dust temperature vs. radius, varying infrared indices of refraction</td>
<td>55</td>
</tr>
<tr>
<td>4-15</td>
<td>Dust temperature vs. radius, varying solar-band indices of refraction</td>
<td>55</td>
</tr>
<tr>
<td>4-16</td>
<td>Variation of optical constants at solar-band wavelengths.</td>
<td>56</td>
</tr>
<tr>
<td>4-17</td>
<td>Dust temperatures for different compositions</td>
<td>57</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>4-18</td>
<td>Indices of refraction for different compositions........................................57</td>
<td></td>
</tr>
<tr>
<td>4-19</td>
<td>Sensitivity of dust temperature to ground temperature.................................58</td>
<td></td>
</tr>
<tr>
<td>4-20</td>
<td>Sensitivity of dust temperature to total optical depth..................................59</td>
<td></td>
</tr>
<tr>
<td>4-21</td>
<td>Sensitivity of dust temperature to surface reflectivity..................................60</td>
<td></td>
</tr>
<tr>
<td>4-22</td>
<td>Sensitivity of dust temperature to $r_{\text{eff}}$ and $v_{\text{eff}}$ in size distribution..........................61</td>
<td></td>
</tr>
<tr>
<td>4-23</td>
<td>Number density vs. altitude, $K=1\times10^7 \text{cm}^2/\text{s}$.................................62</td>
<td></td>
</tr>
<tr>
<td>4-24</td>
<td>Sensitivity of dust temperature to vertical diffusivity..................................62</td>
<td></td>
</tr>
<tr>
<td>4-25</td>
<td>Sensitivity of dust temperature to mixing ratio vs. altitude............................63</td>
<td></td>
</tr>
<tr>
<td>4-26</td>
<td>Effect on temperature of exclusion of CO$_2$ absorption wavelengths.................65</td>
<td></td>
</tr>
<tr>
<td>4-27</td>
<td>Effect on temperature of excluding vibrational modes.....................................66</td>
<td></td>
</tr>
<tr>
<td>4-28</td>
<td>Effect on temperature of including an accommodation coefficient......................66</td>
<td></td>
</tr>
<tr>
<td>4-29</td>
<td>Effect of dust-gas temperature difference on spectra........................................68</td>
<td></td>
</tr>
<tr>
<td>4-30</td>
<td>Heating rates for the base model caused by dust-gas temperature difference........69</td>
<td></td>
</tr>
<tr>
<td>4-31</td>
<td>Heating rates for the dusty case caused by dust-gas temperature difference........70</td>
<td></td>
</tr>
<tr>
<td>5-1</td>
<td>Magnitude of terms in radiative-conductive balance vs. altitude for $r=0.106 \mu m$ grains..........................................................72</td>
<td></td>
</tr>
<tr>
<td>5-2</td>
<td>Magnitude of terms in radiative-conductive balance vs. altitude for $r=9.417 \mu m$ grains..........................................................73</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

a \quad r_{\text{eff}}, \text{ effective radius}
\text{particle semi-major axis}
\text{scattering coefficient}
b \quad v_{\text{eff}}, \text{ effective variance}
\text{scattering coefficient}
c \quad \text{constant in size distribution}
\text{speed of light}
c_{\text{p}} \quad \text{specific heat (constant pressure)}
d \quad \text{particle diameter}
g \quad \cos \theta, \text{ asymmetry parameter}
\text{Mars gravity}
h \quad \text{Planck constant}
k \quad \text{imaginary index of refraction}
\text{extinction coefficient}
\text{Boltzmann constant}
m \quad \text{complex index of refraction}
n \quad \text{number density of particles}
\text{real index of refraction}
n_{\text{gas}} \quad \text{number density of gas}
q \quad \text{mixing ratio}
r \quad \text{radius}
r_{\text{eff}} \quad \text{effective radius}
r_{\text{m}} \quad \text{mode radius}
t \quad \text{time}
\nu_{\text{th}} \quad \text{thermal velocity}
w \quad \text{scattering coefficient}
x \quad \text{dimensionless size parameter}
y \quad m^*x
z \quad \text{altitude}
A \quad \text{scattering coefficient}
B_{\nu} \quad \text{Planck function}
C_{\text{abs}} \quad \text{absorption cross section}
C_{\text{ext}} \quad \text{extinction cross section}
C_{\text{sca}} \quad \text{scattering cross section}
G \quad \text{geometrical cross-sectional area of particles per unit volume}
H \quad \text{scale height}
K \quad \text{vertical diffusivity}
I_{\nu} \quad \text{intensity}
J \quad \text{mean intensity}
\text{Bessel functions of the first kind}
M \quad \text{mass of the CO}_2 \text{ gas molecule}
P \quad \text{phase function}
P_{\text{C}} \quad \text{power lost of collisions}
Q  efficiency (cross section normalized by geometric cross section)

\( P_n \)  Legendre polynomials

\( S_1 \)  scattering amplitude

\( S_2 \)  scattering amplitude

\( S_v \)  source function

T  temperature

\( \nu_{\text{eff}} \)  effective variance

\( \alpha \)  size distribution constant

accommodation coefficient

\( \gamma \)  size distribution constant

\( \eta \)  molecular viscosity

\( \varepsilon \)  emission coefficient

\( \phi \)  azimuthal angle

\( \kappa \)  opacity

\( \lambda \)  wavelength

molecular mean free path

\( \mu \)  \( \cos(\theta) \)

\( \nu \)  frequency

ratio of diffusion time to settling time

dust mass mixing ratio

\( \pi \)  3.1415926…

angle-dependent Mie function

\( \theta \)  angle from vertical

\( \rho \)  particle density

\( \sigma \)  Stephan-Boltzmann constant

\( \tau \)  optical depth

angle-dependent Mie function

\( \varpi \)  single scattering albedo

\( \omega_0 \)  terminal velocity, bottom layer

\( \xi \)  Ricatti-Bessel function

\( \psi \)  Ricatti-Bessel function

\( \Theta \)  potential temperature

\( \Omega \)  solid angle
1. Introduction

1.1. Why study Mars dust?

An understanding of Mars atmospheric dust, its physical properties, and radiative effects has an impact on spectroscopy, reconstructing the history of the planet, and modeling the current climate. Much of the surface of Mars is covered by dust, frustrating those interested in surface mineralogy [Bandfield et al., 2000; Christensen et al., 2001; Johnson et al., 2002; Christensen et al., 2003], and it is therefore found in the atmosphere in significant quantities. Without an active liquid water cycle there is no rain to wash sediments downstream, no oceans to accumulate sediments, and no oceanic subduction zones to serve as a further sink for sediments, including dust-sized particles.

There is a considerable amount of dust suspended in the Martian atmosphere, given its pressure, during ambient conditions [Read and Lewis, 2004]. When regional, and even global, dust storms develop, dust is responsible for warming the atmosphere relative to the surface [Pollack et al., 1979]. Direct evidence for the occurrence of dust in the current Martian atmosphere can be seen in the reddish color of the atmosphere that is apparent in true-color images from the surface, which is due to dust absorption in the visible dominated by iron oxides (although they are not volumetrically the greatest component).

Due to its absorption and scattering properties, dust complicates models of the Martian atmosphere. The physical properties and radiative effects of atmospheric dust are the greatest sources of uncertainty in radiative transfer models of the Mars atmosphere. [Read and Lewis, 2004]. Dust is particularly important to the Martian climate since major dust storms dominate changes in circulation and temperature. A crucial factor in
constraining climate models that attempt to look back in time [Haberle, 1998] is a better understanding of the Martian atmosphere in the present. Understanding dust properties is a crucial hole in a more complete understanding of the Martian climate since it is proportionally more important than in the terrestrial case, where the atmosphere is more complicated chemically and structurally.

Dust is intimately tied up with the entire Mars climate change debate. Combinations of changes in the tilt, orbit, and precession of the planet over time would have led to varying insolation, changing climate conditions [Ward, 1974]. The theory of periodic climate change was first used to explain the existence of polar layered terrains [Kieffer and Zent, 1992], that consist of alternating darker and lighter layers that may contain varying amounts of dust and ice. These layers are thought to contain information about past climate cycles, given the amount of time it would take for a layer of dust to accumulate at current rates [Cutts, 1973].

The polar-layered deposits were first detected in images from Mariner 9 [Soderblom et al., 1973]. A given pair of dark and light layers has a consistent thickness of about 10 to 50 m. Layers can also be traced for tens of kilometers, and may extend farther [Thomas et al., 1992]. While currently it is hypothesized that the net transport of material during major dust storms is to the North [Pollack et al., 1979; Newman, 2001, PhD thesis, quoted in Read and Lewis, 2004, p215], the layered deposits are found near both poles.

If the majority of Martian history was much as it is today, after rapid early degassing and with an inactive interior, it is clear that dust would have been an important
material in the atmosphere for most of Mars history. Except, perhaps, during an early warm, wet period [e.g. Pollack et al., 1987; Squyres and Kasting, 1994; Carr, 1999; Craddock and Howard, 2002], Mars would always have been extremely dry. And it is not clear that Mars ever experienced an extended period of humidity if it lost any primordial atmosphere by 4.45Ga [Gaidos and Marion, 2003].

Clearly dust becomes an important factor on such a dry planet. But even if dry periods were punctuated by transient periods of more recent climate change [e.g. Carr, 1982; Baker et al., 1991], atmospheric dust would have been a factor during that climate change. Dust may have played a role with atmospheric feedbacks during the transition periods, and furthermore it might have remained significant during transient “wet” periods, which could still be quite dry by Earth standards.

As obliquity changes, changes in insolation on various parts of the planet may lead to an increase in atmospheric pressure [Toon et al., 1980; Pollack et al., 1987; Gulick et al., 1997]. If more solar radiation were concentrated on the poles, they would be heated, releasing a reservoir of volatiles (CO$_2$ and H$_2$O) into the atmosphere [Toon et al., 1980]. Once atmospheric pressure and surface temperature begin to increase, other CO$_2$ reservoirs might add further to the atmosphere, such as CO$_2$ in groundwater or adsorbed on clay particles in the regolith [Baker et al., 1991].

Dust also has a potential positive feedback role to play in climate change. Once atmospheric pressure begins to increase, dust storms could become more frequent; then more dust might be transported and deposited on the polar caps [Toon et al., 1980], lowering their albedo. Combined with the effect of more dust in the atmosphere warming
the atmosphere, more CO$_2$ could move from the poles and other reservoirs to the atmosphere [Toon et al., 1980; Baker et al., 1991].

It is also worth noting that at lower obliquity, when the tilt of the planet’s axis is less extreme than today (which is 25.19°, similar to the current obliquity of the Earth, 23.43°), atmospheric pressure may be even lower than the current 6 mbar. In that case, perhaps dust could not be suspended in significant quantity.

Dust transport trends and past obliquity are clues to where water ice may be present today, perhaps preserved under a dust mantle that inhibits sublimation. Such a preserved dust layer has been invoked to explain potential mid-latitude glaciers [Shean et al., 2005; Forget et al., 2006] and hypothesized gully-forming, snow packs on pole-facing crater walls [Christensen, 2003]. For dust to play the role of preserving frozen water below, it would need to be active during the transition periods of climate change, before atmospheric pressure drops too much for ice and snow to remain stable at the surface.

Williams and Toon [2006] suggest that for a ice-protecting dust mantle to prevent sublimation on Mars today it would need to be a meter thick and have been deposited over the course of five years during the last obliquity cycle. For that to be the case, past rates of dust deposition would need to be significantly greater than today, or even that calculated for deposition in polar layered deposits: 500µm/yr [Lasker et al., 2002]. These studies show that the role of dust in Mars climate is very interrelated to questions about the water cycle over Mars history, both because of direct interaction between dust and
water ice and because of the effects of dust on the process of climate change that could have allowed some form of water to be stable at the surface in the past.

1.2. Dust in Martian weather

On today’s Mars dust makes itself evident in the atmosphere with major planet-encircling dust storms every few years [Zurek and Martin, 1993; Leovy, 2001]. Several major dust storms have been monitored when satellites were in orbit capable of recording infrared spectra, notably in 1976 recorded by Mariner 9 [Conrath, 1975], the 1977 storms observed by Viking [Zurek, 1982] and the 2001 planet-encircling dust storm observed by the Mars Global Surveyor (MGS) satellite [Cantor et al., 2001; Smith et al., 2002].

Most studies to date have focused on major dust storm conditions. But there are lesser quantities of dust that continually inhabit the atmosphere during ambient conditions [Pollack, 1977; Toon et al., 1977]. While the formation and development of regional and global dust storms is important to study directly, the study of non-dust storm conditions may actually say more about the climate that leads to such spectacular examples of dust in the atmosphere.

Finally, by studying the extremely dusty environment of Mars we could learn more about the radiative effects of mineral dust in the atmosphere that could also be applied to the Earth’s atmosphere. On Earth it is impossible to study dust in isolation from the other atmospheric constituents with complicated radiative effects in the atmosphere. On Mars there are radiative effects of water ice to be separated from those of dust. Also, water may condense around dust grains in some conditions as it does in our atmosphere [Barker, 1976], making the two harder to separate. Still, these are many
fewer constituents and processes than we find influencing radiation transfer in our own atmosphere-ocean system.
2. Background

This project will use a 1-D radiative-conductive equilibrium model to investigate Mars atmospheric dust temperature. Dust temperatures that vary from the atmospheric gas temperature could have an effect on other studies that involve atmospheric dust, including general circulation models of current and past Mars climate, and other radiative transfer approaches that attempt to derive physical properties of the dust. The potential temperature disequilibrium between atmospheric gas and dust has not been previously incorporated in Mars atmospheric models, and constitutes new information on the relationship between dust and atmospheric temperature useful for climate models.

This section provides a review of basic radiation transfer theory, scattering by atmospheric particles, and numerical approaches to radiative transfer solutions, together with a review of the relevant past work on Mars atmospheric dust, to provide the context for the present study.

2.1. Basic concepts in radiation transfer

In the visible portion of the electromagnetic spectrum, where the intensity of the Sun is a maximum, radiation balance begins with solar energy absorbed, scattered, and transmitted through the atmosphere. The transmitted energy may subsequently be absorbed or reflected by the surface. The energy that is absorbed warms the surface and the atmosphere. In the thermal infrared, surface materials and atmospheric dust emit radiation. The atmospheric dust also absorbs and scatters part of this radiation emitted by the planet.

The radiative transfer equation, subject to relevant assumptions, will balance the above factors. In a simplified differential form it indicates that the change in intensity
with altitude is equal to the intensity, attenuated by an extinction coefficient \((k)\), plus an emission coefficient \((\varepsilon)\), or for a given frequency \((\nu)\):

\[
\cos \theta \frac{dl_v}{dz} = -k_v I_v + \varepsilon_v. \quad (2.1)
\]

The \(\cos(\theta)\) factor arises with the assumption of a plane-parallel atmosphere. Rays may be emitted at any angle with respect to the surface, but in order to enjoy the simplicity of a 1D coordinate system, we choose only to consider the components of those rays perpendicular to the surface (as though the surface were flat). The angle is measured from the normal to the surface.

Next, to facilitate some useful substitutions, the entire equation can be divided by the extinction coefficient \((k)\), which is just the opacity divided by the density

\[
\cos \theta \frac{dl_v}{k_v dz} = -\frac{I_v}{k_v} + \frac{\varepsilon_v}{k_v}. \quad (2.2)
\]

We will define the source function as

\[
S_v = \frac{\varepsilon_v}{k_v}, \quad (2.3)
\]

and the optical depth \((\tau)\) as

\[
d\tau_v = -k_v dz. \quad (2.4)
\]

The optical depth makes the equation easier to manipulate because it is not a function of altitude, but results can be converted back into that coordinate system. The negative sign is used because we choose as a convention that \(\tau=0\) at the “top” of the atmosphere, which for practical purposes is the elevation of the spacecraft making the measurements.
Finally, the substitution

$$\mu = \cos \theta$$  \hspace{1cm} (2.5)$$

will simplify the equation. The result of these substitutions is

$$\mu \frac{dI_v}{d\tau} = I_v - S_v.$$  \hspace{1cm} (2.6)$$

To better approach the problem, Eq. 2.6 can be converted into integral form. A factor of $e^{-\tau/\mu}$ is used to manipulate the equation so that one can solve for $I_v$:

$$\mu e^{-\tau/\mu} \frac{dI_v}{d\tau} = e^{-\tau/\mu} I_v - e^{-\tau/\mu} S_v.$$  \hspace{1cm} (2.7)$$

Note that it also is true that

$$\frac{d}{d\tau} [I_v e^{-\tau/\mu}] = e^{-\tau/\mu} \frac{dI_v}{d\tau} - \frac{e^{-\tau/\mu}}{\mu} I_v.$$  \hspace{1cm} (2.8)$$

So dividing by $\mu$ and combining Eqs. 2.7 and 2.8, one gets

$$\frac{d}{d\tau} [I_v e^{-\tau/\mu}] = -\frac{S_v}{\mu} e^{-\tau/\mu}.$$  \hspace{1cm} (2.9)$$

Then to solve for $I_v$, both sides can be integrated. For outgoing radiation, for example, one integrates between $\tau'=\tau$, which is the total optical depth, and $\tau'=0$. The symbol $\tau'$ is a variable of integration.

$$\int_{\tau}^{0} \frac{d}{d\tau} [I_v e^{-\tau/\mu}] d\tau' = -\int_{\tau}^{0} S_v e^{-\tau'/\mu} \frac{d\tau'}{\mu}.$$  \hspace{1cm} (2.10)$$

$$I_v(0) e^{-\tau/\mu} - I_v(\tau') e^{-\tau'/\mu} = -\int_{\tau}^{0} S_v e^{-(\tau'-\tau)/\mu} \frac{d\tau'}{\mu}.$$  \hspace{1cm} (2.11)$$
Move the $I_\nu(0)$ term to the other side and divide by negative one. Finally, one obtains the general solution

$$I_\nu = I_\nu(0)e^{-\tau_0/\mu} + \int_0^\infty S_\nu e^{-(\tau_\nu-\tau_0)/\mu} \frac{d\tau_\nu}{\mu}$$

(2.12)

for intensity as a function of optical depth.

### 2.2. Scattering

In the case of a purely absorbing, non-scattering atmosphere, the source function ($S_\nu$) can be approximated by the Planck function, $B_\nu$, for blackbody emission. In this case, $B_\nu$ can be integrated and the equation can be solved.

When scattering is considered, the source function has a term that is affected by the proportion and direction of light that is scattered. The scattering term depends on the incident intensity and a phase function ($P$) to describe the direction and intensity of scattered radiation, integrated over a hemisphere. The source function also has a term for blackbody emission by particles, which depends on the proportion of radiation they have absorbed ($1-\sigma$). Including these two terms, the source function can be written

$$S_\nu = \frac{\sigma}{4\pi} \int_0^1 \int_{-1}^1 I_\nu P(\mu,\phi) d\mu d\phi + (1-\sigma)B_\nu$$

(2.13)

where $\sigma$ is the single scattering albedo, or the fraction of extinction due to scattering.

Extinction that is not due to scattering ($1-\sigma$) is due to absorption.

The phase function, $P(\mu,\phi)$, contains information on the extent of scattering as a function of direction and can be approximated in several ways. The approximation for Raleigh scattering is commonly used when the scattering particles are much smaller than
the wavelengths of radiation being used to observe the atmosphere. Raleigh scattering is just a special case of Mie scattering, which can treat any combination of particle size and wavelength. A Mie scattering approach is only necessary when the particle sizes and wavelengths are similar. For large particles, compared to the wavelength, geometric optics is sufficiently accurate.

2.2.1. Single particle Mie Theory

Mie theory of particle scattering and absorption is derived directly from electromagnetic theory. It can be solved exactly for spheres and other ideal particles. All that is required is the complex indices of refraction of the dust grain material.

Sets of functions are found to be solutions to the wave equation that describe the potential normal modes of an electric field around the surface of a sphere. These are the Legendre polynomials \( P_n^m \) and spherical Bessel functions. They are used to calculate coefficients, which lead to scattering amplitudes and cross-sections \( (C) \) eventually applied to a radiative transfer solution.

Those cross sections have units of cm\(^2\), and the extinction cross-section \( (C_{ext}) \) is directly related to the extinction coefficient \( k_\nu \) by:

\[
k_\nu = \sum nC_{ext}
\]

where \( n \) is the number density (particles/cm\(^3\)). If divided by the geometrical cross section \( (\pi r^2) \) the result is usually referred to as an efficiency \( (Q) \). The single-scattering albedo mentioned in Eq. 2.13 is

\[
\omega = \frac{C_{sca}}{C_{ext}}.
\]

(2.15)
Also of note is the absorption cross section $C_{\text{abs}}$, which is the part of the extinction cross section ($C_{\text{ext}}$) that is not due to scattering. That ratio $C_{\text{abs}}/C_{\text{ext}}$ is

$$\frac{C_{\text{abs}}}{C_{\text{ext}}} = 1 - \sigma$$

or simply,

$$C_{\text{abs}} = C_{\text{ext}} - C_{\text{sca}}.$$  \hspace{1cm} (2.17)

Finally, scattering amplitudes and the scattering cross-section (see Appendix A) can be used to calculate the asymmetry parameter, $g = \cos \theta$. When $g=0$ scattering is completely isotropic and when $g=1$ the particles are completely forward scattering.

Everything needed to solve the radiative transfer equation in a two-stream approximation (see Section 2.3) derives from these single-scattering quantities. See Appendix A for Mie theory details.

### 2.2.2. Mie scattering over multiple spheres

Single scattering by multiple particles of different sizes is modeled with a particular particle size distribution of number density as a function of radius. The single sphere quantities must be calculated for each particle radius, weighted by the number density and extinction coefficient, and averaged. A typical function used to represent the particle size distribution is a gamma distribution like the one suggested by Deirmendjian [1969]:

$$n(r) = cr^\alpha e^{-\left(\frac{r}{r_m}\right)^\gamma}$$  \hspace{1cm} (2.18)

Here $c$, $\alpha$, and $\gamma$ are constants, $r$ is the radius and $r_m$ is the mode radius.
2.3. Numerical Approaches to Radiation Transfer

This section will begin with a discussion of the types of radiative transfer approximations appropriate for the present study in the infrared and solar-band portions of the spectrum. Then a brief discussion follows comparing various single and multiple scattering methods, concluding with the specific approaches used here.

In the present study scattering is relatively unimportant in the infrared under most conditions, because almost all extinction by dust is due to absorption at those wavelengths. In this case the single scattering albedo is small. Some studies use infrared solutions that include scattering to extract information on the physical properties of the dust particles [Toon et al., 1977; Erard et al., 1994; Clancy and Lee, 1995; Wolff and Clancy, 2003; Wolff et al., 2006]. Such studies can constrain the size distribution and indices of refraction of the dust.

For the purposes of the present study scattering in the infrared can be ignored because of the focus on relatively low-dust conditions. Even with a low single scattering albedo, scattering can be more important in cases in which a great deal of dust is suspended in the atmosphere, increasing the likelihood that a photon will be scattered at least once on its way through the atmosphere. Sometimes even then it is ignored [e.g. Smith et al., 2000] because it makes it possible to do a survey of a great deal of data without more costly computations.

In the visible part of the spectrum, it is necessary to include scattering because the single scattering albedo is higher at those wavelengths, so proportionally more of the
extinction comes from scattering. But since we focus on relatively clear conditions, a single-scattering (rather than full multiple-scattering) approximation will be sufficient.

Single scattering approximations are simpler solutions that were especially favored when computational resources were not as great as they are today. They assume that radiation scattered in all directions can be treated as though it travels in two streams (up and down normal to the planet’s surface), with a constant factor used to compensate for components in other directions. It is assumed that no photon is scattered more than once in its path through the atmosphere. Two-stream approximations replace the integral in the source function with one of various equations that include the parameters of single scattering albedo and asymmetry parameter, calculated with Mie Theory. Meador and Weaver [1980] summarize different two-stream approximations in planetary atmosphere applications, defining a two-stream approximation as one that can be written, with some manipulation, in the form

$$\frac{dI^+}{d\tau} = \gamma_1 I^+ - \gamma_2 I^- - \pi F \sigma_0 \gamma_4 e^{-\tau/\mu},$$

$$\frac{dI^-}{d\tau} = \gamma_2 I^+ - \gamma_1 I^- - \pi F \sigma_0 \gamma_4 e^{-\tau/\mu}. \tag{2.20}$$

The $\gamma$ factors depend on the dependence of intensity with $\mu$, an assumption that varies with the particular two-stream approximation. The value $\pi F$ is the incident flux.

There are also various approaches to multiple-scattering solutions, which take into account multiple scattering directions and multi-layered atmospheres. A discrete-ordinates method assumes a discrete number of layers and streams at an angle from zenith. It approaches an exact solution as the number of streams at different angles
increases. There are variations on this and other approaches, including Monte Carlo
calculations, summarized by Hansen and Travis [1974] that produce similar results.

One method on which Hansen and Travis focus is a doubling-adding approach, in
which there are two streams and at each layer all radiation is treated as though it must be
transmitted or reflected. This approach allows one to consider multiple scattering, with a
faster solution than the discrete ordinates and other similar approaches. But although it
includes multiple layers it is not as accurate as a discrete-ordinates approach that
considers multiple angles of scattering as well.

Similar to a doubling-adding code is the two-stream, multi-layer solution of Toon
et al. [1989], which is used for the solar-band calculations in this study. Toon et al.
generalize the two-stream solutions, as parameterized by Meador and Weaver, into
multiple layers, so that the radiation field is solved for simultaneously at all layers. This
is a good method for a radiative equilibrium study such as this because it provides the
needed accuracy of including scattering at multiple levels, with a relatively simple, fast
approach.

Solutions like a discrete-ordinates or Monte Carlo approach are the most accurate,
and with today’s computing capabilities would not be too time-consuming to run for a
problem such as that in the present study. As discussed, a multi-layer two-stream
approach is sufficient for the solar-band wavelengths in the present study and is
significantly simpler to implement. It is also more similar to what is used in many larger
models, such as GCMs that require very rapid radiative transfer codes, since that is only
one of many components of the computations. In that case some of the more accurate methods are still too computationally expensive.

2.4. Past Work

Studies of dust in the atmosphere tend to focus on the planet-encircling dust storms that capture attention and so distinguish Martian weather [Toon et al., 1977; Akabane, 1992; Cantor et al., 2001; Smith et al., 2001]. Many of the early conclusions have not been much improved. Pollack et al. [1979] calculated mean particle size, studied the effects of dust on temperature structure, and investigated the processes of dust loading and dissipation, including nucleation of ice around dust particles.

2.4.1. Overview: Radiative Transfer Studies

Geologists are interested in the dust composition, since it is the same dust that covers much of the planet’s surface. The first major work to look at composition is Toon et al. [1977] who modeled Martian atmospheric dust with several terrestrial analogues. They chose spectra from the dust storm of 1971-72 which was observed with the Infrared Interferometer Spectrometer (IRIS) instrument on Mariner 9 upon its arrival at Mars. The inputs to their two-stream single-scattering solution are dust optical depth, composition, and size distribution. The composition depends on the indices of refraction of the material. It is not possible to get an exact fit to the IRIS spectra without knowing the composition of the dust. The sample that produced the best fit was a montmorillonite clay sample. Although some have interpreted the Toon et al. [1977] conclusion to be that the dust is composed of montmorillonite, the authors themselves explain that their work is only a starting point, meant to put constraints on composition and size distribution.
Recent work on Martian atmospheric dust has been driven by a combination of new data and new computing capabilities. In 1991, using Viking Infrared Thermal Mapper (IRTM) data, Clancy and Lee used broad solar-band emission phase function (EPF) observations to confirm Viking lander results for dust size and scattering parameters. EPFs are series of observations of the same area on the surface with the spacecraft positioned at varying look angles off from nadir. Viking data were the latest available in 1991, but computing had advanced to allow a multiple scattering radiative transfer model to be employed. Clancy and Lee used a discrete-ordinates method based on Stamnes et al. [1988], now the freely available DISORT (Discrete Ordinate Radiative Transfer) package.

Clancy and Lee found a greater single scattering albedo and lower asymmetry parameter than that determined by data from the Viking landers and a mean particle size as much as an order of magnitude smaller than that found by Toon et al. [1977]. They also considered multiple locations separately, in addition to making global conclusions about dust properties as most subsequent studies have done. Their 1995 study [Clancy et al., 1995] incorporated Mariner 9 and Phobos data with a reanalysis of the Viking IRTM EPF data. They modeled the dust composition with a palagonite sample, less crystalline than Toon et al.’s montmorillonite and thus more spectrally nondescript. They also showed that a much smaller mode radius (<0.02µm compared to Toon et al.’s 0.4µm) can fit the data, but they did not do an exhaustive study of many size distributions to find a best fit.
Clancy et al. [2003] used EPFs from the Thermal Emission Spectrometer (TES) to determine optical depths, dust size, and scattering parameters. Size distribution of $r_{\text{eff}}=1.5\pm0.1\mu\text{m}$ is consistent with previous results [e.g. Chassefiere et al., 1992; Pollack et al., 1995; Clancy et al., 1995; Tomasko et al., 1999], with the addition that variation in particle size is observed over season and location. For example, a larger effective radius ($r_{\text{eff}}=1.8$-2.5$\mu$m) is found in the Southern hemisphere during the 2001 dust storm. Variation in the dust size could be due to a changing relative size distribution as larger dust grains can remain suspended in the atmospheric conditions of a major storm.

2.4.2. Deriving Indices of Refraction

Snook [1999] demonstrated a method to derive the indices of refraction that provide a best fit to the data, instead of using indices of real materials measured in the lab. Her code iteratively produces fits to the measured spectra, adjusting the optical constants each time in an effort to minimize the error. But Hamilton et al. [2005] have shown that the use of those results to retrieve the individual mineral constituents is difficult. Hamilton et al. used Snook’s optical constants to construct model emission and reflectance spectra that are similar to the end member spectra that exist in libraries for deconvolution of surface mineralogy. They then performed a deconvolution as though it were a measured spectrum of the surface, but were unable to achieve an acceptable match. The lack of success may be a measure of the accuracy of the optical constants that can be derived with Snook’s method for the purposes of satisfying interest in dust mineralogy. The other possibilities are that the mineral components of the Mars dust are
not represented in existing libraries or that the optical constants still need to be further refined.

Nonetheless, sets of optical constants as Snook derived provide a better fit to measured spectra than any terrestrial analog material in the literature. Snook’s rms error is 1.1K (in terms of brightness temperature of the model vs. the measurement), compared to 2.5K for the best alternative developed by Forget et al. [1998]. The Toon et al. [1977] montmorillonite sample has an rms error of 3.2K, by Snook’s measurement. Using “synthetic” optical constants like those derived by Snook minimizes the inaccuracy due to composition for those interested in other dust properties. Like Snook, Forget et al. [1998] tried to avoid the composition question by finding optical constants that lead to modeled spectra that fit the measurements, without regard to whether those optical constants match those of any combination of actual materials. Forget et al. [1998] began by modifying the montmorillonite constants from Toon et al. [1977], but not as systematically or successfully as Snook.

Wolff and Clancy [2003] did a similar analysis using the Clancy et al. [1995] palagonite composition as a set of starting values. As with Snook [1999] and Forget et al. [1998] they try to match infrared observations from orbit only. The Wolff and Clancy result was improved by Wolff et al. [2006], in which TES nadir-looking IR observations were combined with simultaneous Mars Exploration Rover (MER) Mini-TES observations of the atmosphere, to give the most complete look at atmospheric dust to date. The observations from the surface are more influenced by aerosols higher in the
atmosphere; combining the two geometries there is more resolution over altitude in the atmosphere.

2.4.3. Size Distribution

Results for effective radius of the particle size distribution from different studies vary from 0.709\(\mu\)m [Santee and Crisp, 1993] to 2.75\(\mu\)m [Toon et al., 1977]. This variation may have to do with the optical constants used. Variation may also have as much to do with real temporal and spatial variation in the environment as the different methods, data, and assumptions of the different investigators, since Clancy et al. [2003] found variations with season and latitude.

Results from the Mars Exploration Rovers’ atmospheric imaging [Lemmon et al., 2004] show a cross-section weighted mean radius of 1.52 \(\mu\)m above the Gusev site and 1.47\(\mu\)m above the opportunity site. Lemmon et al. also determined that the dust is well mixed to a altitude of 20 to 30km above the surface. Upward-pointing observations cannot be used to determine the distribution higher than these altitudes [Lemmon et al., 2004]. Wolff et al. [2006] also found that mean particle size over the two rover landing sites is different, with values of 1.3 and 1.8\(\mu\)m over the respective rovers. One difference from Lemmon et al., is the Wolff et al. use of altitude dependent dust mixing ratios in the model.

2.4.4. Mixing Ratio

Wolff et al. [2006] used altitude dependent dust mixing ratios following the formulation of Conrath [1975]. Wolff et al., however, treated the ratio of diffusion time
to settling time ($\nu$) as a retrievable parameter, the same for all particle sizes. Most parameters in Conrath’s approximation (see Section 3.2) do not vary spatially or temporally. There is a decrease in dust content with altitude, and the altitudes to which it extends depends mostly on the vertical diffusivity, which depends on three-dimensional weather conditions. Besides local weather, dust distribution could vary over the planet due to extreme topography as well. There is evidence that on the slopes of volcanoes there is an overall increase in dust-CO$_2$ mixing ratio with altitude [Erard et al., 1994].

Including a decreasing mixing ratio with altitude is clearly more realistic for most cases than maintaining the same quantities of dust in suspension near the surface and high in the atmosphere. Using limb observations from the Russian Phobos 2, Moroz et al. [1994] derived an altitude dependence for extinction coefficient, which indicates that mixing ratio, and therefore quantity of dust, does indeed decrease with altitude.

It is also likely that different size particles would have different mixing ratio functions with altitude. That is indeed the case for Mars. Chassefiere et al. [1995] used the solar occultation measurements from Phobos 2 to derive a decreasing mean radius (which remains approximately between 2 and 1µm) and the expected decrease in overall number density of particles. Chassefiere et al.’s measurements only probed the part of the atmosphere between 15 and 25km, where such solar occultation measurements could be made given the Phobos 2 orbital geometry.

The particle size-dependent reality of atmospheric dust distribution continues to be absent from the starting assumptions of complex atmospheric radiative transfer models. Part of the reason it has usually been ignored is that most of the measured data
utilized has involved a nadir-viewing geometry and it is difficult to separate the contributions to extinction and scattering for different layers, especially when there are many other free parameters [as in Wolff et al., 2006]. However, particle size dependent mixing ratios have been experimented with in Mars General Circulation Models [Murphy et al., 1993].

Murphy et al. [1993] used size-dependent dust mixing ratios in predictions of dust transport during the development of a major dust storm. They found that the size distribution would change while dust was transported from a Southern hemisphere source to the Northern Viking lander latitudes. Because models such as that of Murphy et al. predict that spatial variation in size distribution exists due to atmospheric dynamics, it should not be surprising that Lemmon et al. [2004] and Wolff et al. [2006] found that the mean size of dust particles above the MER landing sites, at two different locations on the planet, is slightly different. It seems that the size distribution is partly a function of sorting in transport. It may, similarly, also have to do with variation in the dust size distribution on the ground locally.

In the future size-dependent vertical distributions may be taken into account more often when more extensive data is made available. Multi-spectral Thermal Emission Spectrometer (TES) data, collected on the Mars Global Surveyor (MGS) spacecraft, in varying limb geometries could be used to calculate dust content with altitude to create a data set with greater spatial and seasonal resolution for altitude-dependent size distribution. SPICAM (Spectroscopy for the Investigation of the Characteristics of the
Atmosphere of Mars) on Mars Express also has the potential to measure vertical dust distribution using a stellar occultation technique [Bertaux et al., 2005].

2.4.5. Global Climatology

Smith et al. [2001] used the wealth of data provided by TES to create maps of global climate-related parameters over time. Their dust and water-ice opacity maps were created by calculating the optical depth of a purely absorbing atmosphere, and estimating the percentage due to different absorbers based on their typical spectral shapes. As they point out, for dust that requires two assumptions: that dust is “well-mixed” vertically with CO₂ and that the dust is non-scattering. A further implied assumption is that the CO₂ is at the same temperature as the dust. The Smith et al. results can serve as a starting point because they are tabulated over the entire planet over the duration of the MGS mission. Wolff and Clancy [2003] point out that the Smith et al. results are offset from their opacity results (which differ because they include the effects of scattering) by a predictable amount.

\[
\frac{\tau}{\tau_{\text{TESteam}}} = \frac{\tau_{\text{extinction}}}{\tau_{\text{absorption}}} = \frac{1}{1 - \omega}
\]

(2.21)

where \(\tau_{\text{TESteam}}\) is a value from Smith et al., which are available through NASA’s Planetary Data System (PDS). Using a typical single scattering albedo for Mars dust of \(\sim 0.2\) at 1075cm⁻¹, gives a factor of 1.3 for \(\tau/\tau_{\text{TESteam}}\), which Wolff and Clancy [2003] argue can be consistently used.

Although the results from Smith et al. [2001] don’t include the effects of scattering by atmospheric dust, the relative temperature and opacities of dust and water
ice by latitude and season are useful for understanding the larger climate picture. With the addition of THEMIS (Thermal Emission Imaging System) data, similar data retrievals are performed using an instrument that observes at a different local time [Smith et al., 2003], providing some resolution on diurnal variation as well.

### 2.4.6. Dust in GCMs

The treatment of dust in the approximations of Mars General Circulation Models (MGCMs) varies. Most use a rapid two-stream radiative transfer solution, with varying parameterizations for dust vertical distribution, loading, and dissipation, in an attempt to predict the behavior of major dust storms. Forget et al. [1999] use one of the Toon et al. [1989] results for the infrared, calculating parameters only for two broad bands, and a different doubling-adding code for the solar bands. The dust distribution is represented by the Conrath et al. [1975] equations for dust mass mixing ratio, which are also used in the present study (see Section 3.1).

The Forget et al. model extends above 80km, although it becomes more difficult to interpret above 50km. Interestingly the temperatures they predict in that part of the atmosphere are greater than remote sensing results indicate. This is possibly an inconsistency that could be resolved by taking into account true dust temperatures, but there are other poorly constrained factors: the effects of vertically-propagating waves that strongly link the upper and lower atmospheres, and uncertain upper boundary conditions.

### 2.4.7. Heating Rates

Based on rapid radiative transfer approximations, GCMs need to calculate heating rates, which in turn effect convection and dynamics. It is well understood that dust plays
a major role in heating the atmosphere, especially when it is present in great quantities
[Gierarsch and Goody, 1971]. Atmospheric temperatures during dust storms are warmer
and more uniform throughout the atmosphere [Smith et al., 2002]. Gierarsch and Goody
modeled this behavior before there was as extensive of a dataset on atmospheric
temperature as we have today. They did not investigate if a dust-gas temperature
different would persist, or over what time scale.

Calculations of dust-gas temperature differences have been done for the Earth’s
atmosphere at higher altitudes [Fiocco et al., 1975] and the lower 60km [Fiocco et al.,
1976]. Besides general studies of aerosol temperatures, some focus specifically on water
ice particles in the upper atmosphere [Grams and Fiocco, 1977; Espy and Jutt, 2002]
with implications for the altitudes and atmospheric temperatures at which condensation
takes place.

2.5 Objective of This Study

The present study will explore atmosphere-dust temperature differences in the
ambient, non-dust storm case. We wish to determine whether dust and gas temperature
are in equilibrium in the Mars atmosphere. If not, then the next question is at what
critical pressure dust and gas temperature become decoupled. The final result will be
profiles of dust temperature with altitude for varying particle sizes. We wish to see the
effect that different dust temperatures would have spectroscopically. Finally, we will
comment on the potential implications for other types of studies such as those using
MGCMs.
3. Method

To determine dust particle temperatures in the Martian atmosphere, we start with atmospheric gas temperature profiles which are collected by lander missions descending to the surface of Mars as well as calculated from orbiter data. A radiative-conductive equilibrium is calculated for the atmosphere, and dust grain temperatures are adjusted until there is a balance between all of the rates of energy exchange.

The equilibrium calculation used to find dust particle temperatures in the Mars atmosphere takes into account energy gained and lost by dust grains due to radiation and also energy lost due to collisions with atmospheric CO$_2$ molecules. The later is expected to dominate the dust temperature at the lower altitudes. Where and to what extent the dust and gas become decoupled is the information sought.

3.1. Atmospheric Temperature Profiles

Thermal Emission Spectrometer (TES) temperature profiles [Conrath et al., 2000] are the primary source of input CO$_2$ temperature profiles for the calculations. Their benefit is that they exist for a wide spatial and seasonal sampling. However, even with limb observations they do not extend above about ~70 km in the atmosphere.

TES temperature profiles are calculated based on the shape and depth of the 15µm CO$_2$ absorption in the infrared spectra. The results of these retrievals are tabulated in the PDS for nadir observations, and Mike Smith kindly provided tables of the limb observation-derived profiles as well, which correspond to higher altitudes. The nadir-observation derived temperatures have about a 3K error, while the limb-observation based temperatures have as much as a 7K error.
Examples of temperature profiles from clear and dusty periods are shown in Figure 3-1. The mean optical depth for the clear-period profile at 1075cm\(^{-1}\) (the wavelength of maximum absorption) is 0.1768, and for the averaged dusty temperature profile it is 1.608. The optical depth at 1075cm\(^{-1}\) will be referred to as the reference optical depth. The principal effect of dust on atmospheric temperature can be seen in these profiles: when it is dustier the ground temperature is lower and the temperature profile is warmer and more isotropic in the lower atmosphere.

Several temperature profiles from landers are used for comparison because they extend to greater altitudes than the TES limb temperature profiles can measure. The lander entry profiles reveal structure in the upper atmosphere, as well as the fluctuations of a particular moment.
The Pathfinder temperature profile [Magalhaes et al., 1999] is for a nighttime landing late in Northern summer. It is very near the Viking I site, which passed through the atmosphere during Martian mid-summer. Viking 2 landed during early Martian spring, and about 20° North of the other too. The Viking profiles [Seiff and Kirk, 1977] are plotted with the Pathfinder temperature profile in Figure 3-2.

3.2. Dust Property Inputs

Other inputs to the present dust temperature model include assumptions about dust composition, particle size distribution, and mixing ratio with altitude. The radiative transfer calculations require a surface reflectivity and temperature for boundary conditions. The outputs are an infrared spectrum at the “top” of the atmosphere that can be compared to observations and dust temperature for each particle size considered at each altitude in the grid. Table 3-1 summarizes the variables.
Table 3-1 Parameters that go into the present model, and the output expected. See Table 3-2 for specific input values used in the base model.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature profile</td>
<td>Solar-band and IR model spectra</td>
</tr>
<tr>
<td>Dust material indices of refraction</td>
<td></td>
</tr>
<tr>
<td>Size distribution</td>
<td>Dust particle temperatures at every layer and for every particle size</td>
</tr>
<tr>
<td>Mixing ratio with altitude</td>
<td></td>
</tr>
<tr>
<td>Reference optical depth</td>
<td></td>
</tr>
<tr>
<td>Surface temperature</td>
<td></td>
</tr>
<tr>
<td>Surface reflectivity</td>
<td></td>
</tr>
</tbody>
</table>

The actual composition of the particles is one of the great unknowns in the overall Martian dust equation. It must be assumed, based on previous work in the literature, and input into the Mie calculations of dust scattering properties. The indices of refraction, the second input in Table 3-1, reflect the particle composition. The indices can be measured for specific real substances, or fit to best match the data. Here we use indices of refraction determined by Wolff et al. [2006] as a set of starting values. We vary them by 10% and also use some indices measured in the lab for particular minerals (enstatite and quartz) for comparison. These minerals are not likely candidates for Mars dust composition, but they provide an estimate for the effect of optical constants on the modeled temperature.

Another variable, particle size distribution, is based on the formula from Hansen and Travis [1974], which is a modified gamma distribution commonly used in Mars atmospheric models, and similar to the example in Eq. 2.19:

\[ n(r) = \text{const} \times r^{(1-3b)/b} e^{-r^{1ab}} \]  

(3.1)
where \( a \) is the effective radius, and \( b \) is the effective variance of the distribution. The values used for the base model are \( r_{\text{eff}}=1.5\mu\text{m} \) and \( v_{\text{eff}}=0.2\mu\text{m} \), in line with the consensus on overall size distribution. These too are varied to determine the relative effect of the input on the final temperature results.

The effective radius is calculated using:

\[
 r_{\text{eff}} = \frac{\int_{r_1}^{r_2} r \pi r^2 n(r) dr}{\int_{r_1}^{r_2} \pi r^2 n(r) dr} = \frac{1}{G} \int_{r_1}^{r_2} r \pi r^2 n(r) dr 
\]

where \( G \) is the geometric cross-sectional area of particles per unit volume. The effective variance is:

\[
 v_{\text{eff}} = \frac{1}{Gr_{\text{eff}}^2} \int_{r_1}^{r_2} (r - r_{\text{eff}})^2 \pi r^2 n(r) dr 
\]

The constant in Eq. 3.1 is determined based on the reference optical depth, given for the “clear” and dusty averaged cases in Section 3.1. The distribution is first calculated for \( \text{const}=1 \), and later the appropriate factor is found by which to multiply the distribution so that there is an appropriate amount of dust to lead to the reference optical depth specified. A size distribution based on Eq. 3-1 is shown in Figure 3-3.
Figure 3-3 Size distribution at the lowest atmospheric layer for the base model.

Mixing ratio vs. altitude is allowed to vary for different particle sizes. Therefore, the size distribution at different altitudes will not be the same. Allowing size distribution to vary with altitude in a radiative transfer calculation is one of the features of this study that differs from other radiative transfer focused approaches. The mixing ratio with altitude is determined from the equations used by Conrath et al. [1975]:

\[ q(z) = q_0 e^{\nu(1 - e^{z/H})} \]

The distribution is Figure 3-3 is the input for particle size near the surface \( q_0 \). In Eq. 3.4 \( \nu \) is the ratio of diffusion time to settling time, and is given by:

\[ \nu = \frac{H}{K} \omega_0^* \]

where \( H \) is the scale height (H), \( K \) is the vertical diffusivity, and \( \omega_0^* \) is the terminal velocity at the bottom layer. The terminal velocity \( \omega_0^* \) is given by:
\[ \omega_0^* = \left( \frac{\rho_D g d^2}{18 \eta} \right) \left( 1 + 2 \frac{\lambda}{d} \right) \]  

(3.6)

where \( \rho_D \) is the particle density, \( g \) is Martian gravity, \( \eta \) is molecular viscosity, \( \lambda \) is molecular mean free path for the CO\(_2\) gas, and \( d \) is the particle diameter. The values used in Eqs. 3.4 - 3.6 for the base model are given in Table 3-2.

**Table 3-2** Base model values used for the variables in the vertical particle distribution.

<table>
<thead>
<tr>
<th>Mixing ratio parameter:</th>
<th>Base model value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle density (( \rho_D ))</td>
<td>3 g/cm(^3)</td>
</tr>
<tr>
<td>Mars gravity (g)</td>
<td>370 g/cm(^2)</td>
</tr>
<tr>
<td>Molecular viscosity (( \eta ))</td>
<td>(1.5 \times 10^{-4}) g/(s⋅cm)</td>
</tr>
<tr>
<td>Molecular mean free path (( \lambda ))</td>
<td>(1.38 \times 10^{-3}) cm</td>
</tr>
<tr>
<td>Scale height (H)</td>
<td>10 km</td>
</tr>
<tr>
<td>Vertical diffusivity (K)</td>
<td>(10^6) cm(^2)/s</td>
</tr>
</tbody>
</table>

The result for these particular values, using the base model size distribution from Eq. 3.1 as \( q_0 \) in Eq. 3.4 is shown in Figure 3-4.

![Figure 3-4](image-url)  

**Figure 3-4** Number density vs. altitude. Shows ten different particle sizes, using base model values (Table 3-2).
Table 3-3 Base model, clear atmosphere, input values.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Base model value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indices of refraction</td>
<td>Wolff et al.(2006)</td>
</tr>
<tr>
<td>Size Distribution</td>
<td>Hansen &amp; Travis(1974), ( r_{\text{eff}}=1.5\mu\text{m}, v_{\text{eff}}=0.2\mu\text{m} )</td>
</tr>
<tr>
<td>( q[z] )</td>
<td>Conrath(1975), see above</td>
</tr>
<tr>
<td>Reference optical depth (at 1075cm(^{-1}))</td>
<td>0.1768</td>
</tr>
<tr>
<td>Ground Temperature</td>
<td>260.82K</td>
</tr>
<tr>
<td>( \text{CO}_2 ) temperature profile</td>
<td>Average TES clear, as above</td>
</tr>
<tr>
<td>Ground reflectivity (avg. albedo)</td>
<td>0.15</td>
</tr>
<tr>
<td>Solar incidence angle</td>
<td>34°</td>
</tr>
</tbody>
</table>

Table 3-3 summarizes all of the specific values used in the base model, which is based on values accepted in the literature and reasonable averages. The base model represents ambient, non-dust storm conditions, which are relatively clear. When we refer to the “clear” case however, there is still some dust in the atmosphere, causing it to be of interest.

The variables are summarized again in Figure 3-5, which also shows a chart of the algorithm for calculating dust temperatures. The following sections will explain the radiative transfer approaches used, and how the radiative-conductive equilibrium is calculated.
Figure 3-5  Flow chart of calculation. Illustrates the process of calculating dust grain temperatures, with simplified version of Eq. 3.12.
3.3. Radiative Transfer

The calculations are done with a series of programs written in the Interactive Data
Language (IDL), sometimes based on other’s work as noted. The radiation fields in the
solar bands and in the infrared are calculated separately using different radiative transfer
approximations. Both produce output of mean intensity (I) for all of the altitudes under
consideration. The mean intensity is the angle-averaged intensity, or:

\[ \int I d\Omega = 4\pi J \]  

(3.7)

The solar-band radiation field is found using the method of Toon et al. [1989, see
section 2.3]. An IDL version can be found in Appendix B, along with the other programs
written for the present study.

To demonstrate the accuracy of my version of the solar-band radiative transfer
code, it is compared to the results shown in Toon et al. [1989] for five test cases as
outlined in Table 3-4. The Toon et al. results and those obtained here for the test cases,
are both compared to a more accurate discrete-ordinate solution in Table 3-5. The results
obtained here (based on the Toon et al. method) are extremely consistent with the results
that they present.

For the infrared, the Schwarzschild-Milne equations are used to calculate the
infrared mean intensity at each layer [Chandrasekhar, 1960], working upward from the
ground. This approach does not include scattering, but includes the effects of absorption
and reemission at each layer.
Table 3-4 Parameters for test cases. Based on Toon et al. [1989], Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>1</td>
<td>0.9</td>
<td>0.9</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>$g$</td>
<td>0.794</td>
<td>0.794</td>
<td>0.794</td>
<td>0.848</td>
<td>0.848</td>
</tr>
</tbody>
</table>

Table 3-5 Error comparison. Shows percent errors of Toon et al. [1989] in the first column under each case number, and my percent errors from the exact results in the second columns ("t.s." for "this study").

<table>
<thead>
<tr>
<th>$\tau$ \case</th>
<th>1 Toon</th>
<th>t.s.</th>
<th>2 Toon</th>
<th>t.s.</th>
<th>3 Toon</th>
<th>t.s.</th>
<th>4 Toon</th>
<th>t.s.</th>
<th>5 Toon</th>
<th>t.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau=0$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.3</td>
<td>0.36</td>
<td>0.3</td>
<td>0.21</td>
<td>5</td>
<td>4.6</td>
<td>0</td>
<td>4.8</td>
</tr>
<tr>
<td>$\tau/20$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.3</td>
<td>0.36</td>
<td>0.5</td>
<td>0.34</td>
<td>5</td>
<td>4.6</td>
<td>0</td>
<td>6.1</td>
</tr>
<tr>
<td>$\tau/10$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.3</td>
<td>0.32</td>
<td>0.7</td>
<td>0.58</td>
<td>5</td>
<td>4.7</td>
<td>1</td>
<td>7.7</td>
</tr>
<tr>
<td>$\tau/5$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.2</td>
<td>0.29</td>
<td>1</td>
<td>1.2</td>
<td>5</td>
<td>4.9</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>$\tau/2$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.1</td>
<td>0.12</td>
<td>1</td>
<td>3.1</td>
<td>5</td>
<td>5.2</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>$3\tau/4$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.1</td>
<td>0.04</td>
<td>5</td>
<td>4.8</td>
<td>5</td>
<td>5.4</td>
<td>80</td>
<td>83</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.2</td>
<td>0.24</td>
<td>6</td>
<td>6.0</td>
<td>5</td>
<td>5.5</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

3.4. Radiative-conductive Equilibrium

The radiative-conductive equilibrium balance includes terms for the energy that is gained and lost by dust grains. The terms for absorption of energy by dust grains are based on the solar and infrared mean intensity fields calculated with radiative transfer approximations (see Section 3.3). Mean intensity ($J$) is integrated over wavelength for each layer, resulting in the rate of power absorption:

$$4\pi \int_{\lambda} Q_{abs} J d\lambda$$

(3.8)
The mean intensities are weighted by the absorption efficiencies, which are functions of wavelength and particle size. The factor of $4\pi$ comes from the definition of mean intensity (Eq. 3.7).

The radiation emission term mirrors the radiative energy absorption terms. The grains are assumed to emit like blackbodies, so the Planck function $B_\lambda$ is used to represent emitted intensity. The absorption efficiency is used to weight this term because blackbodies absorb and emit with equal efficiency at a given wavelength.

The final term, which deals with the energy lost in collisions between dust grains and gas molecules, is the conduction part of the balance. It can be written to first order as:

$$\frac{5}{2} k(T_a - T_{\text{gas}}) \ v_{th} n_{\text{gas}}$$

Eq. 3.9

The two parts of Eq. 3.9 are the energy exchanged between the gas molecules and grains, $\frac{5}{2} k(T_a - T_{\text{gas}})$, and the collision rate, $v_{th} n_{\text{gas}}$, calculated from the velocity of the gas molecules times their number density, where the velocity is

$$v_{th} = \sqrt{\frac{8kT}{\pi M}}$$

Eq. 3.10

and M is the mass of the CO$_2$ gas molecule. The subscript ‘a’ indicates the semi-major axis of a particular dust size.

The entirety of the term for energy lost through conduction is not represented in Eq. 3.9. The energy part of the term in Eq. 3.9 includes a factor of $\frac{1}{2}$ for each degree of freedom: three translational degrees of freedom and two rotational. The vibrational
modes are not very significant at the temperatures of the Mars atmosphere. But the vibrational effects can be included as well by adding the following to Eq. 3.9:

\[
\sum_{i=1}^{3} \frac{\Theta}{e^{\Theta/T_a} - 1}
\]  \hspace{1cm} (3.11)

where \(\Theta = h\nu/k\) is called a potential temperature, \(\nu\) is frequency and \(h\) and \(k\) are constants.

The quantity is added up for \(\nu\) at each of the fundamental frequencies of the first transition from the unexcited state. For \(\text{CO}_2\) gas these are 1388.3cm\(^{-1}\) for the symmetric stretching mode, 667.3cm\(^{-1}\) for the bending mode, and 2349.3cm\(^{-1}\) for the asymmetric stretching mode. As wavenumber increases they become less significant, but all are included.

Combining all of the terms discussed, the following equation is used to balance the energy gained and lost by the dust grains, and can be solved for temperature using a bracketing-bisection method, where a limiting range is put on particle temperature and then that range is bisected repeatedly until the particle temperature that balances the equation is found.

\[
4\pi \int_{\lambda_{sa}} Q_{abs}(a,\lambda)J_{\lambda}d\lambda + 4\pi \int_{\lambda_{sa}} Q_{abs}(a,\lambda)J_{\lambda}d\lambda = 4\pi \int_{\lambda_{sa}} Q_{emis}(a,\lambda)B_{\lambda}d\lambda
\]

\[
+ \left[ \frac{5}{2}k(T_a - T_{\text{gas}}) + \left( \sum_{i=1}^{3} \frac{\Theta}{e^{\Theta/T_a} - 1} - \sum_{i=1}^{3} \frac{\Theta}{e^{\Theta/T_{\text{gas}}}} \right) \nu_{th}n_{\text{gas}} \right]
\]  \hspace{1cm} (3.12)

The grain size and wavelength dependent values of \(Q_{abs}\) are calculated, along with scattering parameters for the solar-band radiative transfer code, using a code given in Appendix B for Mie theory calculations.
After Eq. 3.12 is solved for $T_a$, a check is performed to see if the temperature varies from the previous value by more than 0.1 Kelvin. If so then the infrared radiative transfer calculation is done again for dust at that temperature, and $T$ is solved for again. This entire process is done separately for different size dust grains and at all of the altitudes under consideration.
4. Results and Analysis

This chapter starts with a review of the results using the parameters of the base model, and then compared to a dustier case. Section 4.3 will show the effect of using different temperature profiles, which allow us to observe theoretical dust temperature higher in atmosphere, testing the consistency of the model in that limit. Section 4.4 is a systematic analysis of parametric uncertainty. What follows then is a discussion of other uncertainties in the model and finally a brief look at spectroscopic effects of higher dust temperatures.

4.1. The Base Model

The temperatures of the dust grains can be calculated using the radiative-conductive equilibrium model described. We start with the parameters of the base model described in Chapter 3, discuss the behavior of the dust temperature profiles, and show that the pressure at which dust and gas temperature decouple is consistent with a simple calculation.

Figure 4-1 shows that as altitude increases, and gas pressure decreases, the dust grain temperature becomes more and more decoupled from the CO$_2$ gas temperature, and consequently more affected by radiative absorption and emission. Generally, the temperature of the larger particles differs the most from the gas temperature at these altitudes, although the pattern also changes even higher in the atmosphere.
Figure 4-1 Particle temperature with altitude. Shows ten different particle sizes (radius is in microns), and the input CO$_2$ gas temperature profile for comparison.

The consistency of the result can be confirmed with a simple calculation of the theoretical pressure at which dust and gas temperatures begin to decouple. The critical pressure can be calculated by balancing heating and cooling terms. Instead of an exact calculation, we approximate each of the radiative terms with

$$\pi \int Q_{abs} J(\lambda) d\lambda = Q_{abs} \sigma T^4$$

as though $Q_{abs}$ did not depend on wavelength, $\lambda$. It is averaged over all wavelengths in the solar-band and IR portions of the spectrum separately. The resulting equation is solved for $n_g$ the number density of gas

$$n_g = \frac{Q_{vis}^{\text{vis}} \sigma T_{\text{top}}^4 + Q_{\text{abs}}^{\text{IR}} \sigma T_{\text{grid}}^4 - Q_{\text{abs}}^{\text{IR}} \sigma T_{\text{dust}}^4}{\frac{5}{2} k(T_{\text{dust}} - T_{\text{gas}}) \cdot \frac{1}{4} v_{\text{th}}}$$

(4.2)

where $T_{\text{top}}$ = the temperature of the grains due to solar heating, in isolation from the atmosphere, and $T_{\text{dust}}$ = $T_{\text{gas}} + 5K$. All other symbols are as in previous equations. The relationship of $n_g$ vs. $T_{\text{gas}}$ above can be compared with the known relationship between
the two to find an intersection, which is the location of the critical gas density. The results are shown in Table 4-1.

**Table 4-1** Pressure of gas-dust temperature decoupling. Shows the first order theoretical and modeled pressures where dust temperature reaches 5K above gas temperature for varying particle sizes.

<table>
<thead>
<tr>
<th>particle size</th>
<th>theoretical decoupling pressure</th>
<th>modeled decoupling pressure</th>
<th>z, corresponding to modeled pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.106µm</td>
<td>0.004 mbar</td>
<td>0.014 mbar</td>
<td>66 km</td>
</tr>
<tr>
<td>0.136µm</td>
<td>0.009 mbar</td>
<td>0.020 mbar</td>
<td>62 km</td>
</tr>
<tr>
<td>0.209µm</td>
<td>0.014 mbar</td>
<td>0.031 mbar</td>
<td>57 km</td>
</tr>
<tr>
<td>0.369µm</td>
<td>0.034 mbar</td>
<td>0.060 mbar</td>
<td>50 km</td>
</tr>
<tr>
<td>0.710µm</td>
<td>0.049 mbar</td>
<td>0.086 mbar</td>
<td>46 km</td>
</tr>
<tr>
<td>1.409µm</td>
<td>0.104 mbar</td>
<td>0.114 mbar</td>
<td>43 km</td>
</tr>
<tr>
<td>2.713µm</td>
<td>0.218 mbar</td>
<td>0.166 mbar</td>
<td>39 km</td>
</tr>
<tr>
<td>4.780µm</td>
<td>0.379 mbar</td>
<td>0.199 mbar</td>
<td>37 km</td>
</tr>
<tr>
<td>7.329µm</td>
<td>0.456 mbar</td>
<td>0.218 mbar</td>
<td>36 km</td>
</tr>
<tr>
<td>9.417µm</td>
<td>0.549 mbar</td>
<td>0.240 mbar</td>
<td>35 km</td>
</tr>
</tbody>
</table>

The theoretical decoupling pressures are consistent with the results. Larger particles begin to increase in temperature over the gas temperature at lower altitudes. The altitude is really determined by the gas density, which correlates with pressure as in an ideal gas.

**4.2. “Clear” vs. Dusty Atmospheres**

The “clear” base model can be compared with a dustier case. The dusty case, like the base model, is based on an average of TES temperature profiles, this time for those dusty periods that also have almost zero atmospheric water ice opacity and low water vapor column abundance (≈4.8 pr µm). The corresponding average optical depth at the reference wavelength is used for the reference optical depth (τ=1.608), and the
corresponding average incidence angle (43°) and the corresponding average ground temperature ($T_{\text{grd}} = 291.9$K) for those observations are used. All else remains the same as the TES averaged clear case. The comparison is shown in Figure 4-2, where temperatures are plotted only for the smallest and largest particle sizes for both cases for simplicity.

Figure 4-2 Clear vs. dusty case particle temperatures. Shown are dust temperatures for the smallest and largest grain sizes for the clear base model, as in Figure 4-1, and the dusty case.

For both the clear and dusty cases, up to about 1 mbar there is negligible difference between dust and gas temperature for the largest particles. For the smallest particles, the difference is negligible up to almost 0.1 mbar. As 0.01 mbar is approached the grain temperatures in both the clear and dusty cases approach one another, because the more that radiative effects dominate the temperature, the more they approach a consistent set of values for a given dust composition.
Figure 4.3 Dust-gas temperature difference. The differences between CO$_2$ gas temperature and dust grain temperature for two particle sizes in both the clear and dusty cases.

The effect of dust temperatures approaching the same values at altitude, regardless of the gas temperature, is also seen in the temperature differences. They are almost the same for both cases because the gas temperature profiles, while structurally different, also approach a similar value at the greatest height shown. But it is notable that the difference between dust and gas temperature is less for the dusty case than the clear case. Although gas temperature is not allowed to vary in this model, the dust will actually heat the gas when there is more suspended dust present, warming the atmosphere instead of the other way around.

4.3. Varying Temperature Profile

Next several other temperature profiles, which were measured by lander missions as they passed through the Mars atmosphere, will be used. These are useful because they extend to greater altitudes and allow us to examine the dust-gas temperature difference at higher altitudes and lower pressures. At greater altitudes there may be negligible
quantities of dust particles, making the following theoretical temperatures less important to the GCM models, but they still provide a check on the reasonableness of temperatures calculated at the upper limit. Currently, however, the altitude to which dust is found under different conditions is not very well constrained. Viking images of the limb were used to determine that a dust haze existed up to an altitude of 60km [Jaquin et al., 1986]. If a future mission to sample atmospheric dust such as the proposed Sample Collection for Investigation of Mars (SCIM) were to collect dust at those or higher altitudes, it could be expected to have reached the temperatures shown in Figure 4-4.

**Figure 4-4** Dust temperatures with Viking I temperature profile. Dust temperatures calculated for the Viking I lander atmospheric temperature profile, using a reference optical depth of ~0.3, a higher vertical diffusivity (K=1x10^(-7)), and all else as in the TES average clear case in Figure 4-1.
Figure 4-5 Dust temperatures with Viking II temperature profile. The same as Figure 4-4, with a temperature profile measured by the Viking II lander.

The trend that larger particles get warmer does not hold at higher altitudes. The dust temperature profiles asymptote to the radiative equilibrium temperature for a grain of that size and composition at Mars distance from the Sun, in isolation from any atmosphere. The shape of the atmospheric gas temperature profile does not matter once the particles reach higher altitudes where they become completely decoupled from the gas. It bears mentioning again, however, that even with the higher vertical diffusivity used, there are effectively zero particles at those highest altitudes.

The temperature of a given particle size at the highest altitude has a constant value for a given set of optical properties even if other inputs are allowed to vary. In that limit, at the lowest pressures, the effect of interactions with the atmospheric gas are negligible and temperature is controlled only by radiative absorption and emission.

The entire dust temperature profiles can be produced as though there were no atmospheric gas (Figure 4-6). For a given particle size, the temperature at the upper layers changes little throughout the atmosphere. There is an influence of surface
radiation warming the lower layers, making them warmer. Both solar and infrared radiation face extinction through the atmosphere because there is a finite optical depth. Although intensity of solar-band radiation is larger, particles absorb more efficiently in the infrared. It is just as though the atmosphere doesn’t affect dust temperature.

![Graph](image)

**Figure 4-6** Dust temperature in isolation from gas. The Pathfinder gas temperature profile with dust temperature profiles for different radii (in microns) when dust temperature is not allowed to be affected by interactions with atmospheric gas. There is solar radiation and infrared heating from the surface.

In the case where radiative and conductive terms are all taken into account (as in Figures 4-4 and 4-5), dust temperature is in equilibrium with gas temperature until 20-50km, depending on particle size. Then dust and gas temperature begin to decouple, with dust grains warmer than the surrounding gas. In the mid-altitude regime, between ~30-60km, larger grains are warmer than smaller grains. At those altitudes conduction is no longer the dominant term, but it still contributes more to cooling than emission. Conduction is not strongly particle-size dependent, therefore the relative temperatures of different particle sizes is controlled only by how efficiently they absorb radiation.
At the highest altitudes (>~65km), the largest particles are not the hottest. In the high-altitude regime conduction is negligible, and it can be predicted which particles are warmest based on their relative absorption efficiencies in the solar-bands and infrared. In both wavelength regions the average absorption efficiency approaches one for large particles. The smaller particles absorb less efficiently, and therefore also emit less efficiently. But the difference between the efficiencies in the visible and infrared determine the temperature that a particular size grain can reach. See Figures 4-7 and 4-8.

**Figure 4-7** Absorption efficiency vs. radius in the visible and infrared. Difference between the curves correlates with radii that achieve warmer and cooler temperatures.
Figure 4-8 Maximum dust temperature vs. radius. Dust grain temperatures reached with complete decoupling from the atmospheric gas, as a function of radius.

Using either of the Viking temperature profiles, or the Pathfinder one, the identical result of Figure 4-8 is produced, reinforcing the confidence in the algorithm. The data for dust temperature at the “top” of the atmosphere follow the trends described above. We see that the peak temperature is reached around 0.4µm. There is also a local low around 5µm, where the most efficient cooling is achieved \((Q_{\text{abs}}\text{ in the infrared at a maximum})\).

Toward the large particle limit grains should reach the blackbody temperature (226K) if \(Q_{\text{ex}}=1\). The maximum temperature calculated is in fact the blackbody temperature when all \(Q_{\text{ex}}\) are forced to equal one, with a single scattering albedo of zero. This is an important test of the accuracy of all the parts of the algorithm. Figure 4-9 shows that the blackbody temperature is calculated within 8K for this limiting case. This is an acceptable error given that the input gas temperature profiles from TES have error of 4-7K.
Figure 4-9 Case of dust at blackbody temperature. The temperature profile when $Q_{ex}=1$ and there is no influence from the surface or atmospheric gas on temperature.

It is instructional to look again at temperature differences. For the Pathfinder entry profile, for example, they could be as high as 289K for the warmest particles. The 5µm or so coolest particles would be as much as 172K warmer than the gas, if there were any particles at those altitudes, which is unlikely. The maximum temperature difference does not necessarily occur at the highest altitude, because while the dust grain temperatures smoothly asymptote at low pressures, the CO$_2$ gas temperature may still fluctuate a great deal.
**Figure 4-10** Dust temperatures with Pathfinder temperature profile. As in Figures 4-4, 4-5 but for the Pathfinder entry temperature profile.

**Figure 4-11** Dust-gas temperature differences for lander temperature profiles. Temperature differences between 0.369μm particles and the atmospheric gas for Viking I (red), Viking II (blue) and Pathfinder (black) atmospheric CO\textsubscript{2} temperature profiles.

The difference that a different temperature profile makes on the temperature of the dust relative to the atmospheric gas is small. For example in Figure 4-11, where temperature differences may appear extreme, at 60km the difference is no more than 87K for the largest particles, not much more than in Figure 4-3. Because dust can be found at that altitude this result is quite significant.
4.4. Parametric Uncertainty Analysis

Starting from the base model of Section 4.1, this section will show the relative effect of varying each of the input parameters (see Table 3-1). We will see how uncertainties in indices of refraction, ground temperature, optical depth, surface reflectivity, and particle distribution propagate to uncertainty in dust particle temperatures. Then we can determine to which parameters the results are most sensitive, indicating where future work should concentrate.

4.4.1. Indices of Refraction

The indices of refraction of the dust make the largest difference in the modeled dust temperatures. First we will show the effect of varying the Wolff et al. indices by 10%. Then we will use completely different compositions, although they are not at all realistic, to better grasp the effects of optical properties on dust temperature.

Starting with the optical constants derived by Wolff et al. [2006], 10% is both added and subtracted at all wavelengths at once. The real and imaginary indices are altered separately, although their values are related to one another in reality.
Figure 4-12  Sensitivity of dust temperature to indices of refraction. Temperature profiles for dust grains at r=0.106µm (left cluster) and r=9.417µm (right cluster), with indices altered as shown in the legend.

It can be seen that a 10% error makes a bigger difference in the temperatures of the larger particles. The imaginary component (k) influences how much the grains absorb. When it is increased, dust particles absorb more and get hotter, and the opposite when k is less. The real indices have an opposite effect, as shown in detail for all particle sizes in Figure 4-13.

Figure 4-13  Dust temperature vs. radius, varying indices of refraction. Illustrates the same result as Figure 4-12, over all radii at 60km.
While increasing n decreases the temperature at high radii, it increases the temperature by a lesser amount at smaller radii. Consider the approximate relationship between $Q_{abs}$ and the complex index n in the Rayleigh small particle limit. There

$$Q_{abs} \approx \frac{(m^2-1)}{(m^2+2)}.$$ 

If $m=n-i$, then $Q_{abs} \approx \frac{(n^2-2in)}{(n^2-3in+3)}$. So since $n<1$, an increase in n results in an increase in $Q_{abs}$, which leads to warmer particles. This is the main behavior seen in Figure 4-12, but the overall behavior of $Q_{abs}$ is dependent on n and k at the same time.

It’s also interesting to see how much a 10% error in the infrared indices affects the temperatures compared to the same percent error in the solar-band indices. Figures 4-14 and 4-15 show that the solar-band indices have a slightly greater effect on the modeled temperatures.

The relative importance of the solar-band optical constants speaks to the importance of not focusing on a single wavelength region of the spectrum. Taking into account dust temperature, which affects how many particles are required to produce a certain amount of emission, requires simultaneously considering the solar wavelengths.
Figure 4-14 Dust temperature vs. radius, varying infrared indices of refraction. Temperatures for all particle radii at 60km when only infrared optical constants are altered by 10%.

Figure 4-15 Dust temperature vs. radius, varying solar-band indices of refraction. Temperatures for all particle radii at 60km when only solar-band optical constants are altered by 10%, accounting for the vast majority of the error in Figure 4-12.
Figure 4-16 Variation of optical constants at solar-band wavelengths. Optical constants from Wolff et al., (2006) plus 10% (red) and minus 10% (blue) for the solar-band region, which produces a greater change in particle temperature.

Next, the particle temperatures that result from using entirely different dust grain compositions are considered. One is derived from an enstatite sample [Jager et al., 2003] and the other is quartz [Henning and Mutschke, 1997]. Neither of these examples is hypothesized to represent the actual Mars atmospheric dust composition, but they demonstrate the importance of this parameter for models including dust.

Once again we see that changes in the optical constants make a bigger difference in the temperatures of the larger particles. The enstatite optical constants yield temperatures less than the Wolff et al. constants, which are altered from the palagonite sample of Clancy et al. [1995]. For the smaller grain size shown, in an atmosphere of enstatite dust, dust and gas temperature would be in equilibrium at least up to 70km. The quartz optical constants yield slightly higher temperatures.
Figure 4-17 Dust temperatures for different compositions. Results with optical constants for enstatite and quartz compared with the base model which uses indices derived in Wolff et al. [2006]. The normal-weight solid lines are for \( r=0.106 \mu \text{m} \) and the dashed are for \( r=9.417 \mu \text{m} \).

Figure 4-18 Indices of refraction for different compositions. Indices for the base model, Wolff et al. [2006] in black, enstatite in red, and quartz in green, over all wavelengths.

4.4.2. Ground Temperature, Optical Depth, and Reflectivity

Boundary conditions such as surface temperature and reflectivity, as well as the total quantity of dust in the atmosphere as reflected in the optical depth, are treated as inputs in the present model. It would be possible to treat ground temperature and optical depth as retrievable parameters when trying to match orbital data. The following
sensitivity analysis, similar to what was done for indices of refraction, doesn’t reflect how some inputs would then be interrelated.

Changing ground temperature has a small effect on the dust grain temperature results. An error of 10K produces a change of less than 6K in the modeled atmospheric dust temperature for the largest particle, as shown in Figure 4-19.

Changing optical depth by 10%, since it is small to begin with, has little effect. Figure 4-20 shows the effect of changing the reference optical depth by an order of magnitude, also an unrealistically high error for the parameter. The total amount of dust in the atmosphere has little effect on the dust grain temperatures. They are somewhat cooler when there is more total dust in the atmosphere because more energy from the planet is absorbed by grains closer to the ground, before it reaches the altitudes where dust temperature depends primarily on radiation.

**Figure 4-19**  Sensitivity of dust temperature to ground temperature. Shows the result of altering the ground temperature in the base model by +/-10K to 301.9K (red) and 281.9K (blue). Left group of profiles is for r=0.106µm and right group is for r=9.417µm.
Finally the surface reflectivity is an input that might have an impact on the final temperature results. If calculations of albedo are accurate there should not be too much error in this parameter in general, but since many values in data retrievals are interdependent, it may. For the base model we have used an average albedo for Mars of 0.15. Upper and lower values shown in Figure 4-21 are an order of magnitude lower, and approaching one respectively. When reflectivity is greater, more solar energy reflects off the surface instead of being absorbed, so it can be absorbed by the atmospheric dust grains and make them hotter. The opposite is true for a lower reflectivity.
Figure 4-21  Sensitivity of dust temperature to surface reflectivity. Shows the base model (in black), surface reflectivity=0.015 (in blue), and reflectivity=0.915 (in red).

Surface emissivity is not considered in this model. Since it varies locally and the base model for this study is an average anyway, it is always assumed equal to one. For future data retrievals, perhaps to derive improved optical constants, it is not certain what the relative importance of surface emissivity would be compared to dust temperature and the inputs discussed here.

4.4.3. Dust Distribution

The last remaining input parameters to consider are the dust distribution parameters; both the base distribution and the way it is mixed vertically must be considered. For the lowest level in the atmospheric model, a size distribution n(r) is specified based on an effective radius and effective variance. The base model values are $r_{\text{eff}}=1.5\mu\text{m}$ and $v_{\text{eff}}=0.2$. The consensus from previous work [e.g. Chassefiere et al., 1992; Pollack et al., 1995; Clancy et al., 1995; Tomasko et al., 1999], is an effective radius approximately between 1 and 2μm. These inputs to the size distribution, along
with a wider variance $v_{\text{eff}}=0.4$ (with the standard $r_{\text{eff}}$) are all shown in Figure 4-22. As one can see, the relative distribution between different size grains changes the modeled grain temperatures little when the total volume of dust in the atmosphere remains constant.

![Figure 4-22](image.png)

**Figure 4-22** Sensitivity of dust temperature to $r_{\text{eff}}$ and $v_{\text{eff}}$ in size distribution. The effect of changing size distribution parameters within currently accepted values. The base model is $r_{\text{eff}}=1.5\mu m$ and $v_{\text{eff}}=0.2$.

The parameter in the vertical mixing ratio function that is least certain is the vertical diffusivity ($K$). The resulting plot of number density vs. altitude for various particle sizes when $K=1\times10^7 \text{cm}^2/\text{s}$, instead of the base model value of $1\times10^6$, is shown in Figure 4-22. The value of $K=1\times10^7$ is used with the lander gas temperature profiles to force dust to be present at higher altitudes.

Using a value of $K=1\times10^7 \text{cm}^2/\text{s}$ in the base model changes the shape of the distribution vs altitude. But in the base model, unlike the models using lander temperature profiles where the same value of $K$ is used, dust is not allowed to exist above 70km. Thus, in order to have the same total optical depth, there needs to be more dust...
below 70km than there is when the model is extended to higher altitudes. That means there is more dust at the higher altitudes considered (~40-60km) relative to the lower altitudes.

**Figure 4-23** Number density vs. altitude, $K=1 \times 10^7 \text{cm}^2/\text{s}$. Shows the number density of different sized dust grains with altitude when $K=1 \times 10^7 \text{cm}^2/\text{s}$. The legend shows the size of dust grains in microns. Compare with Figure 3-4 for $K=1 \times 10^6$.

**Figure 4-24** Sensitivity of dust temperature to vertical diffusivity. The gas temperature profile (far left) and base model (both in black), and the base model altered with $K=1 \times 10^7$ (in red). The left set of dust profiles is for $r=0.106 \mu\text{m}$ and the right set is for $r=9.417 \mu\text{m}$.
A similar effect occurs if the dust distribution is not allowed to vary with altitude, and there is an unrealistic constant quantity of dust at lower and higher altitudes. Then there is a greater relative amount of dust at the higher altitudes compared to the base model, which leads to warmer dust temperatures at those altitudes. If there is a constant mixing ratio function for all particle sizes then the result is similar to that for constant quantity with altitude, because there are more large particles relative to small particles at higher altitudes.

![Figure 4-25](image)

**Figure 4-25** Sensitivity of dust temperature to mixing ratio vs. altitude. Shows the effect of a constant quantity of dust with altitude, and a single mixing ratio function for all particle sizes, on dust temperature compared to the base model for \( r=0.106\mu m \) (left set) and \( r=9.417\mu m \) (right group).

In summary, the input parameters of indices of refraction, surface temperature and reflectivity, total reference optical depth, and dust distribution parameters can all have some effect on the modeled dust temperatures. Most do not cause an error of more than 10K, even with greater than likely errors on the input. The indices of refraction make the biggest difference, and particularly accurate indices in the solar bands are most important to get accurate dust grain temperatures. A large effect also occurs when the relative
distribution of dust with altitude changes. Ground temperature can also make a notable difference. Nothing however should be discounted when it comes to doing retrievals from the data taking into account dust temperature, where each must be distinguished and all are interrelated.

4.5. Other sources of uncertainty

Two other sources of uncertainty in the results have to do with the treatment of the atmospheric CO$_2$ in the energy equilibrium used to calculate temperature. One has to do with the term for infrared energy absorbed by the dust grains. In that spectral range there is a significant CO$_2$ absorption at 15µm, which is not taken into account. It is as though the dust grains absorb energy from the surface at and around 15µm, when really most of it is absorbed by the gas instead.

Energy and temperature for the gas molecules are not allowed to vary in this model; a measured temperature profile is taken as a given. A more complete model might allow gas temperature to vary and model the 15µm absorption and its precise effect on the IR energy available to dust grains. In the absence of that, we estimate an equivalent width (of 3.2µm) for the CO$_2$ absorption and subtract the portion of IR absorption within that wavelength range around 15µm.

Using a smaller value for absorption by particles in the IR makes a difference, as shown below, mostly for the larger particles. As expected the dust temperatures are lower when absorption by CO$_2$ is taken into account. This is just an approximation, however, and future models should fully model the CO$_2$. 
Effect on temperature of exclusion of CO\textsubscript{2} absorption wavelengths. The gas temperature profile (bold) and base model (black) results for \( r=0.106 \mu\text{m} \) (left) and \( r=9.417 \mu\text{m} \) (right), with results adjusted for the portion of IR emission from the surface that's absorbed by CO\textsubscript{2} at 15\( \mu\text{m} \) (red).

The other source of uncertainty relating to atmospheric CO\textsubscript{2} is found in the term for energy lost by dust grains in collision with gas molecules. The energy of the gas is part of the term, including translational, rotational, and vibrational modes. Figure 4-27 shows the result when the vibrational modes, the smallest component, are not included. There is another factor, which we ignore, which can be used to more accurately describe heating by grain-gas thermal exchange, called an accommodation coefficient (\( \alpha \)).

\[ \alpha = \frac{T_2 - T}{T_d - T} \]  
\( (4.3) \)

where \( T \) is the gas temperature, \( T_d \) is the grain temperature, and \( T_2 \) is the grain temperature immediately after a collision [Lequeux, 2005]. The temperature \( T_2 \) has to do with the fact that energy in a collision is not transmitted instantaneously, but it is not possible to determine for collisions of CO\textsubscript{2} with our Martian dust. Figure 4-28 shows the result if an accommodation coefficient of 0.92 [Dickens, 1934] is used in order to
illustrate the magnitude of this effect. The accommodation coefficient value comes from a study where it was measured using a platinum wire in a glass tube of CO$_2$ [Dickens, 1934]. It proves to be an effect on the same order as including the vibrational modes of the CO$_2$ molecules, and both effects are quite small (< 2K).

**Figure 4-27** Effect on temperature of excluding vibrational modes. As in figure 4-25, but with red profiles showing the dust temperatures at two radii when vibrational modes of CO$_2$ are not considered in the energy calculation.

**Figure 4-28** Effect on temperature of including an accommodation coefficient. As in figure 4-26, but with red profiles showing dust temperature profiles when an accommodation coefficient of 0.92 is added.
A thorough treatment of the atmospheric gas would not really be complete without considering the other atmospheric gas constituents, which make up 5% of the atmosphere. A non-local thermodynamic equilibrium (non-LTE) model would properly treat the exchange of energy between all of them and the effect on heating rates, although it is more important higher in the atmosphere. There may be an overlapping level where non-LTE considerations are relevant and dust is also present.

4.6. Effect of dust temperature on infrared spectra

The significant differences between gas and dust temperature in the middle and high altitudes does not have a big effect spectroscopically. This is to be expected since most of the quantity of dust in the atmosphere is located at lower altitudes where it is in equilibrium with gas temperature. In Figure 4-29, solid black lines show the brightness temperature spectra calculated with the dust temperatures the same as gas temperatures for the “clear” (upper) and dusty (lower) cases. The red lines show what the spectra would be if dust were all at its warmest modeled temperature.

In the relatively clear case ($\tau_{\text{dust}}=0.1768$), a small dust absorption is observed but there is no notable difference in the spectra with the new dust temperatures. On the other hand in the dusty case ($\tau_{\text{dust}}=1.608$), there is a difference in the depth of the dust absorption, with the warmer dust producing a shallower absorption. This means that for a given observed dust absorption, there may actually be more dust than previously thought to produce the same depth of a dust feature. The magnitude of this effect, however, would be small because the difference in brightness temperature is less than a degree.
Figure 4-29  Effect of dust-gas temperature difference on spectra. Shows the effect of dust on the spectra when the highest dust temperatures are used (in red) for the (relatively clear) base model (above) and the dustier case (below).

4.7. Effect of dust temperature on heating rates

Even if the dust-gas temperature difference does not have a large effect spectrally, it may still be an important factor to consider in heating rates. Heating rates of atmospheric gas by dust grains are calculated for each grain size and altitude. Those rates are calculated with

\[
\frac{dT}{dt} = 3\nu P_C / 4\pi r^3 \rho_a c_p ,
\]

(4.4)
after Fiocco et al. [1975]. In Eq. 4.4 \(\nu\) is the dust mass mixing ratio for a particular size grain. \(P_C\) is the power lost to collisions:

\[
P_C = 4\pi r^2 \cdot \frac{5}{2} k(T_a - T_g) \cdot \frac{1}{4} n_g v_g
\]

(4.5)
when the vibrational modes aren’t included. The particle density \(\rho_a\) is 3g/cm\(^3\). And the value used for specific heat at constant pressure of the CO\(_2\) gas is 700 J/(kg K).
Using the above formula and values, heating rates are calculated and plotted for the “clear” base model (Figure 4-30) and the dusty case (Figure 4-31).

**Figure 4-30** Heating rates for the base model caused by dust-gas temperature difference.
Heating rates induced by dust-gas temperature differences are small for the “clear” base model: no more than several Kelvin a day, depending on radius and altitude. For the dusty case, however, there are heating rates of upwards of 20 Kelvin per day around 1µm and 30-40km. The largest heating rates occur for a grain size that gets relatively warm, and at altitudes where those grains have a significant mass mixing ratio. Even if those grains get even warmer at higher altitudes it doesn’t lead to a higher heating rate as long as the mass mixing ratio is low. In conclusion, dust-gas temperature difference does have an important effect on heating rates that should be accounted for in time-dependent models of dust storm development.
5. Discussion

This study has demonstrated that dust temperatures in the Martian atmosphere cannot be assumed to be in equilibrium with measured atmospheric gas temperatures, especially at altitudes above \(~40-60\)km. Mars atmospheric models, up to this point, ignore varying dust temperature. In this study particles of different sizes are treated separately, and as expected temperature varies with grain size.

Dust grain temperature is indeed in equilibrium with atmospheric gas temperature at lower altitudes, but it decouples at a predictable, size-dependent altitude becoming hotter with altitude until reaching a limiting temperature. The limiting temperature is the temperature a grain achieves in total isolation from the atmosphere and the surface. It is greater than the blackbody temperature for scattering particles modeled using accepted optical constants \([\text{Wolff et al.}, 2006]\) which absorb more efficiently than they emit.

At intermediate altitude (\(~40-60\)km) temperature increases with grain size. Dust has been observed with limb observations at these altitudes, so the temperature differences are quite significant. At the highest altitudes, where very little dust is present, the limiting temperature is reached. The limiting temperature does not increase with grain size, but exhibits more complex behavior as a function of particle radius.

Limiting temperature decreases with grain size for intermediate size particles. There is a small particle size that attains the maximum temperature, with temperature decreasing again with even smaller particles. There is also a temperature limit to which grain temperature asymptotes in the large particle limit, which is warmer than the blackbody temperature for the particles modeled.
By plotting the magnitude of each of the terms with altitude one can see which energy exchange dominates the temperature for different sized particles. Figures 5-1 and 5-2 show the magnitude of the terms for the smallest and largest particles respectively for the case using the temperature profile from the Viking I lander (see Figure 4-4). In yellow is the magnitude of the quantity $4\pi \int_{\lambda_{v}}^{\lambda_{i}} Q_{\text{abs}}(a, \lambda) J_{\lambda} d\lambda$ for absorption in the solar bands. In orange is the magnitude of the quantity $4\pi \int_{\lambda_{v}}^{\lambda_{i}} Q_{\text{abs}}(a, \lambda) J_{\lambda} d\lambda$ for absorption in the infrared bands. In red is emission in the infrared: $4\pi \int_{\lambda_{v}}^{\lambda_{i}} Q_{\text{emis}}(a, \lambda) B_{\lambda} d\lambda$. The conduction term is divided into two parts, involving the gas temperature

$$\left[\frac{5}{2} k T_{\text{gas}} + \sum_{\theta=1}^{3} \frac{\Theta}{e^{\theta/T_{\text{gas}}} - 1}\right]v_{n} n_{\text{gas}}$$

(in blue), and involving the particle temperature

$$\left[\frac{5}{2} k T_{a} + \sum_{\theta=1}^{3} \frac{\Theta}{e^{\theta/T_{a}} - 1}\right]v_{n} n_{\text{gas}}$$

(in green).

**Figure 5-1** Magnitude of terms in radiative-conductive balance vs. altitude for $r=0.106\mu m$ grains.
The two parts of the conduction terms clearly dominate at lower altitudes in both cases, balancing each other with other terms negligible. When the other terms become comparable in magnitude to the conduction, dust and gas temperature begin to decouple. That decoupling happens earlier for the larger grains where the total magnitude of non-conduction terms is greater at all altitudes. When the emission term becomes significantly greater than conduction (about an order of magnitude) the dust temperature profile begins to asymptote to its limiting temperature based purely on radiative balance. For the case in Figure 5-1 that begins to happen at about 100km, whereas for the largest particle (Figure 5-2) radiative emission begins to dominate cooling at about 60km.

In a sensitivity analysis of the various input parameters, the biggest effect on dust grain temperature comes from the optical constants associated with their presumed composition, in both the solar and infrared regions. Ground temperature is more
important than one might expect, since the infrared absorption term can be larger than the solar-band absorption due to the higher absorption efficiency at those wavelengths. But an even greater effect comes from the relative distribution with altitude. When more particles are found at higher altitudes they shift the dust temperature profiles towards hotter temperatures at those altitudes, but don’t make a significant different at lower altitudes where temperatures first begin to diverge. Properly accounting for absorption in the infrared by CO$_2$ causes an effect of the same order.

All of the above observations will be relevant to Mars atmospheric models that address the potential effects of varying dust temperature. True dust temperature will be more crucial to include in some types of calculations than others.

Varying dust temperature does not have a large effect spectroscopically. During major dust storms, however, it may slightly affect the calculation of dust optical depth, because there is a slight change in the depth of the dust absorption at 1025cm$^{-1}$. As Smith et al. [2001] point out, their opacity results become less accurate under conditions of high dust opacity, and by ignoring scattering they already have errors on the order of ten percent. Their results are more useful for comparing relative opacity under different conditions anyway and the errors are unlikely to change much due to the dust temperature differences described here. Others, who attempt to derive more accurate opacities [such as Wolff et al., 2006] might begin considering this phenomenon, but it is unclear whether the effect of dust temperature would be significant compared to other errors.

Dust-gas temperature differences should not have as large of an effect on other spectroscopically derived quantities such as retrievals of optical constants and size
distribution for the dust itself, the variation of which do more to affect the shape of the broad, minerally indistinct absorption from dust. The temperature differences modeled here only seem only to slightly affect the depth of that absorption, and that only at the dustiest times.

Perhaps much more significant will be the effect of dust-gas temperature differences on heating rates when applied to time-dependent GCMs. The difference in vertical temperature structure between the atmospheric gas and suspended dust particles will affect the heating of the atmosphere as major dust storms develop and more dust becomes suspended in the atmosphere. In the dusty case modeled, particles near 1µm in size at mid-altitudes (30-40km) can cause heating rates upwards of 20K/day. Therefore, the temperature difference calculations done here should be incorporated in Mars General Circulation Models (MGCMs). It should be a relatively rapid calculation which could be incorporated directly instead of being parameterized based on my results.

Such a direct incorporation would indeed be preferable since the dust-gas temperature differences do not represent just another model input to adjust. Complicated feedbacks exist between large-scale dynamical atmospheric motions and the temperature structure. In future work it will be useful to study major dust storm formation in time-dependent models where dust and atmospheric gas temperatures each come to steady state, taking into account radiative heat transfer, conductive heat transfer between gas and dust, as well as convective heat transfer all at once.

Dust grain temperatures significantly hotter than previously assumed could have an effect on other types of calculations as well, including considerations of water ice and
chemical reactions. *Smith et al.* [2003] mention that H$_2$O may condense around dust grains during periods where greater quantities of water ice are observed spectrally.

Additionally, this study assumes a pure CO$_2$ atmosphere although the Mars atmosphere is only 95% carbon dioxide. Not only might water ice condense around dust grains, but, for example, HO$_x$ species might adsorb on dust grains, which by extension could reduce CO oxidation [Anbar et al., 1993], affecting our understanding of Mars atmospheric chemistry.

It is not surprising to find dust temperatures that vary from the temperatures of the gas in which they are suspended. Independently calculated particle temperatures have already been considering in astrophysical and terrestrial atmospheric applications. *Chiang and Goldreich* [1997] model grains in circumstellar disks that are heated above the gas temperature. It is a similar problem to that of planetary atmospheres, as they point out, although there are some major differences. Primarily they point out a much longer settling rate for grains, dust temperature completely decoupled from gas temperature due to lower gas density, and a lack of significant absorption from spectral lines due to the gas. Fundamentally, however, the same physics is at work.

*Fiocco et al.* [1975, 1976] show that temperatures of particles high in the Earth’s atmosphere can become hotter than the atmospheric gas temperature, a consideration that is not overlooked in terrestrial atmospheric models today. As in the present study, the particles are found to be in equilibrium with the gas temperature up to a critical pressure when the temperatures of the particles become hotter, finally asymptoting to a maximum
temperature at altitude. Also similarly they find a particular small particle size (~0.5µm) that attains the highest temperatures.

Dust is not the most obvious application of the Fiocco et al. results for the Earth’s atmosphere, mostly because it is not as ubiquitous. A primary application has been to water-ice aerosols [Grams & Fiocco, 1977; Epsy & Jutt, 2002]. The true temperatures of water-ice aerosols under different conditions have an obvious effect on altitudes for condensation and freezing, affecting clouds and precipitation.

Water-ice aerosols also exist on Mars, so it is possible that this type of temperature equilibrium study could be put to bear on that as well, perhaps together with dust in a study of dust as condensation nuclei. Meanwhile Mars, as a system where dust is the main complication in studies of the atmosphere could help us better understand the radiative effects of suspended dust which are impossible to study in isolation of other Earth atmosphere constituents.

In the end this study prepares us to better tackle larger questions about Mars weather and climate. Demonstrating the importance of real dust grain temperatures to Mars atmospheric science applications is the immediate result. It has also tackled an atmospheric radiation problem with a model that can provide physical intuition that is applicable beyond Mars.
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APPENDIX A

MIE THEORY NUMERICAL SOLUTION
A.1 Numerical Solution for Mie coefficients

First, two angle-dependent functions will simplify the writing of the subsequent equations:

\[ \pi_n(\mu) = \frac{P^1_n(\mu)}{\sin \theta} \]  \hspace{1cm} (A.1)

\[ \tau_n(\mu) = \frac{dP^1_n(\mu)}{d\theta} \]  \hspace{1cm} (A.2)

The above are determined numerically using the Legendre Polynomials \( P^1_n \) and their derivatives [Deirmendjian, 1969]:

\[ \pi_n(\theta) = \cos(\theta) \frac{2n-1}{n-1} \pi_{n-1}(\theta) - \frac{1}{n-1} \pi_{n-2}(\theta) \]  \hspace{1cm} (A.3)

\[ \tau_n(\theta) = \cos(\theta)[\pi_n(\theta) - \pi_{n-2}(\theta)] - (2n-1)\sin^2(\theta)\pi_{n-1}(\theta) + \tau_{n-2}(\theta). \]  \hspace{1cm} (A.4)

The scattering coefficients are defined using the Riccati-Bessel functions \( \psi_n \) and \( \xi_n \). The complex index of refraction is \( m = n + ik \). The dimensionless size parameter is \( x = \frac{2\pi r}{\lambda} \).

The coefficients are

\[ a_n = \frac{\psi'_n(mx)\psi_n(x) - \psi_n(x)\psi'_n(mx)}{\psi_n(mx)\xi'_n(x) - \xi_n(x)\psi'_n(mx)} \]  \hspace{1cm} (A.5)

\[ b_n = \frac{\psi'_n(mx)\psi_n(x) - m\psi_n(x)\psi'_n(mx)}{\psi_n(mx)\xi'_n(x) - m\xi_n(x)\psi'_n(mx)} \]  \hspace{1cm} (A.6)
The quantities above can be calculated numerically through recursion as well, as shown [Deirmendjian, 1969]:

\[
a_n(m, x) = \frac{(A_n(y) + \frac{n}{m}) \text{Re}[w_n(x)] - \text{Re}[w_{n-1}(x)]}{(A_n(y) + \frac{n}{m}) w_n(x) - w_{n-1}(x)},
\]

(A.7)

\[
b_n(m, x) = \frac{(mA_n(y) + \frac{n}{x}) \text{Re}[w_n(x)] - \text{Re}[w_{n-1}(x)]}{(mA_n(y) + \frac{n}{x}) w_n(x) - w_{n-1}(x)},
\]

(A.8)

with \(w_n\) also recursively defined:

\[
w_n(x) = \frac{2n-1}{x} w_{n-1}(x) - w_{n-2}(x)
\]

\[
w_0(x) = \sin(x) - i \cos(x) = \sqrt{\frac{\pi x}{2}} [J_{\frac{1}{2}}(x) + iJ_{-\frac{1}{2}}(x)],
\]

(A.9)

\[
w_{-1}(x) = \cos(x) + i \sin(x) = \sqrt{\frac{\pi x}{2}} [J_{-\frac{1}{2}}(x) + iJ_{\frac{1}{2}}(x)]
\]

where \(J\) are the Bessel functions of the first kind. \(A_n\) is defined instead with downward recurrence as suggested by Bohren and Huffman [1983]:

\[
A_{n-1}(y) = \frac{n}{y} = \frac{1}{A_n(y) + \frac{n}{y}}
\]

(A.10)

where \(y=mx\).

The scattering coefficients and the angle-dependent functions can be used to calculate quantities with physical significance. First, the scattering amplitudes:
\[ S_1 = \sum_n \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n), \quad (\text{A.11}) \]

\[ S_2 = \sum_n \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n). \quad (\text{A.12}) \]

\( S_1 \) and \( S_2 \) are elements of the matrix that defines the relation between incident and scattered electric field amplitudes:

\[
\begin{pmatrix} E_i \\ E_r \end{pmatrix} = \begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix} \cdot \frac{e^{-ikr+ikz}}{ikr} \begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix}, \quad (\text{A.13})
\]

where \( l \) and \( r \) are electric fields parallel and perpendicular to the plane of scattering.

They are used to calculate the asymmetry parameter, which indicates the degree to which scattering is isotropic \((g=0)\) or forward-scattering \((g=1)\):

\[
\overline{\cos \theta} = \frac{\lambda^2}{4\pi m^2} \int_{-1}^1 (|S_1|^2 + |S_2|^2) \mu d\mu. \quad (\text{A.14})
\]

The final useful quantities are the extinction and scattering cross-sections

\[
C_{\text{sca}} = \frac{\lambda^2}{2\pi m^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2), \quad (\text{A.15})
\]

\[
C_{\text{ext}} = \frac{\lambda^2}{2\pi m^2} \sum_{n=1}^{\infty} (2n+1) \text{Re}\{a_n + b_n\}. \quad (\text{A.16})
\]
APPENDIX B

INTERACTIVE DATA LANGUAGE PROGRAMS
Appendix B

B.1 Mie calculations
First, a wrapper for the Mie code that calls it over all angles and calculates the asymmetry parameter (here ‘cbar’) which encapsulates the relative angular distribution of scattering.

pro getbasicmie, radius, Ksc, Kex, lamda, m_lamda, cbar
; - This calculates parameters from Mie theory of particle scattering
; for use in code for thesis temp equilibrium problem.
; - does one radius and one wavelength at a time

numang = 181
angle = dindgen(numang)
cbar = 0.0D
intens1=dblarr(181) & intens2=dblarr(181)

x = (2.D0 * !DPI * radius) / lamda ; x is dimensionless

for q = 0, numang-1 do begin
theta = angle[q]
thetarad = rad(theta)
dthetarad = rad(angle[2]) - rad(angle[1])

omega = 0.D0
intensity = [0.0D, 0.0D]

bign = round((x + x^(0.5D) + 1.D0))
if bign LT 10 then bign = 10

getamps, m_lamda, theta, x, bign, Ksc, Kex, omega, intensity

intens1[q] = intensity[0]
intens2[q] = intensity[1]

cbar = cbar + ((intens1[q]+intens2[q])*cos(thetarad)*sin(thetarad)*dthetarad)
endfor

cbar = cbar * (1.0D/ (Ksc * x^2.0D))
return
end
This code, called by ‘getbasicmie’ above, calculates scattering and extinction efficiencies and scattering amplitudes based on the coefficients collected from calls to ‘getab’ and ‘getangs’.

```plaintext
pro getamps, m, theta, x, n, Ksc, Kex, omega, intensity
   FORWARD_FUNCTION getabnew, getangs

   ab = dcomplexarr(2, n+1) ; two column array of coefficients 'a' and 'b'
   tempang = dblarr(2, n+1)
   S1 = dcomplex(0.0, 0.0)
   S2 = S1
   Kex = 0.D0
   Ksc = Kex
   intensity = dblarr(2)
   xtemp = x

   ab = getab(ab, x, m, n)
   tempang = getangs(n, theta)
   pi = tempang[0,*]
   tau = tempang[1,*]

; calculate the complex amplitudes (S_) and cross-sections (K__) for i = 1, n do begin
   S1 = S1 + (((2.D0*i)+1.D0)/(i*(i+1.D0)))*((ab[0,i]*pi[i])+(ab[1,i]*tau[i]))
   S2 = S2 + (((2.D0*i)+1.D0)/(i*(i+1.D0)))*((ab[1,i]*pi[i])+(ab[0,i]*tau[i]))
   Ksc = Ksc + (((2.D0*i)+1.D0)*((abs(ab[0,i])^2.0D + (abs(ab[1,i]))^2.0D))
   Kex = Kex + ((2.D0*i)+1.D0)*real_part(ab[0,i]+ab[1,i])
endfor

   Ksc = Ksc * (2.D0/(xtemp*xtemp))
   Kex = Kex * (2.D0/(xtemp*xtemp))

   omega = Ksc / Kex ; albedo

; define the intensity functions for single scattering
   intensity[0] = real_part(S1*conj(S1)) ; scattering perpendicular to scattering plane
   intensity[1] = real_part(S2*conj(S2)) ; scattering parallel

return
end
```
Called by ‘getamps’, this code calculates the scattering coefficients (a and b) based on recurrence relationships for the Ricatti-Bessel functions.

function getab, ab, x, m, n
    w = dcomplexarr(n+1)
    A = w
    acoeff = A
    bcoeff = acoeff

    y = m*x

    A[n] = dcomplex(0.D0, 0.D0)
    w[0] = dcomplex(sin(x), -cos(x))
    wneg = dcomplex(cos(x), sin(x))

    for i = n, 1, -1 do begin ;downward recurrence as suggested by BH
        A[i-1] = (i/y)-(1.D0/(A[i]+(i/y)))
    endfor
    A[n] = (-n/y) + (1.D0/((n/y)-A[n-1]))

    for i = 1, n do begin
        if (i NE 1) then begin
            w[i] = (((2.D0*i)-1.D0)/x) * w[i-1]) - w[i-2]
        endif else begin
            w[i] = (((2.D0*i)-1.D0)/x) * w[i-1]) - wneg
        endelse

        acoeff[i] = (((A[i]/m)+(i/x))*real_part(w[i])) - real_part(w[i-1]) /
                      (((A[i]/m)+(i/x))*w[i]) - w[i-1])
        bcoeff[i] = (((A[i]*m)+(i/x))*real_part(w[i])) - real_part(w[i-1]) /
                      (((A[i]*m)+(i/x))*w[i]) - w[i-1])

        ab[0,i] = acoeff[i]
        ab[1,i] = bcoeff[i]
    endfor

    return, ab
end
This function, also called by ‘getamps’ determines the angle-dependent functions (\(\pi\) and \(\tau\)), using recurrence relations for the Legendre polynomials.

\[
\text{function getangs, n, theta}
\]
\[
\text{pi = dblarr(n+1)}
\]
\[
\text{tau = dblarr(n+1)}
\]
\[
\text{answer = dblarr(2, n+1)}
\]
\[
\text{thetarad = rad(theta)}
\]
\[
\text{pi}[0] = 0.D0
\]
\[
\text{tau}[0] = 0.D0
\]
\[
\text{pi}[1] = 1.D0
\]
\[
\text{tau}[1] = \cos(\text{thetarad})
\]
\[
\text{for i = 2, n do begin}
\]
\[
\pi[i] = ((2.D0*i - 1.D0)/(i - 1.D0))*\cos(\text{thetarad})*\pi[i-1]) - ((i/(i - 1.D0))*\pi[i-2])
\]
\[
\text{tau}[i] = (i*}\cos(\text{thetarad})*\pi[i]) - ((i+1.0D)*\pi[i-1])
\]
\[
\text{endfor}
\]
\[
\text{answer}[0,*] = pi
\]
\[
\text{answer}[1,*] = tau
\]
\[
\text{return, answer}
\]
end
This function, called by subsequent routines, does weighted averages of the Mie parameters found above, over a particle size distribution.

function thesimie, radius, n, onemiestruct, Kscarr, Kabsarr, cbarr, logrw

    numr = size(radius)
    numr = numr[1]

    Kexarr = Kscarr + Kabsarr
    omega = Kscarr/Kexarr
    gam1 = (7.0D - (4.0D*omega)-(3.0D*omega*cbarr))/4.0D

    beta_ab = 0.0D
    beta_sc = 0.0D
    beta_ex = 0.0D
    bottom = 0.0D
    omegabar = 0.0D
    cosbar = 0.0D

; get averaged single scattering albedo & asymmetry parameter:
for i = 0, numr-1 do begin
    ; lit way
    beta_ab = beta_ab + n[i]*Kabsarr[i] *(radius[i]^2.0D) * !DPI *logrw[i]
    beta_sc = beta_sc + n[i]*Kscarr[i] *(radius[i]^2.0D) * !DPI *logrw[i]
    beta_ex = beta_ex + n[i]*Kexarr[i] *(radius[i]^2.0D) * !DPI *logrw[i]
    bottom = bottom + n[i]*(radius[i]^2.0D) * !DPI *logrw[i]

    cosbar = cosbar + (n[i]*kscarr[i]*!DPI*(radius[i]^2.0D)*cbarr[i]*logrw[i])
endfor

omegabar = beta_sc / beta_ex
    cosbar = cosbar / beta_sc

onemiestruct.narr = n
onemiestruct.g = cosbar
if (beta_ab/bottom GE 0) then onemiestruct.Qabs = beta_ab/bottom
else onemiestruct.Qabs = 0.0D
if(beta_ex/bottom GE 0) then onemiestruct.Qex = beta_ex/bottom
else onemiestruct.Qex = 0.0D
return, onemiestruct

end
B.2 Radiative Transfer

B.2.1 Pre-processing
This first code sets up arrays for altitude, temperature, pressure, particle size distribution, mixing ratio, wavelength (in the solar-bands), and indices of refraction. It calls the Mie codes above to calculate and tabulated optical properties. It also sets up the dust distribution with altitude, normalized to the appropriate reference optical depth.

Some parameters are changed by hand for different possible inputs. Results are put into an idl ‘save’ file at the end, which is restored in subsequent subroutines.

pro populations

; constants that will be needed
kconst = 1.380658D-23 ; Boltzmann constant (Joules/Kelvin)
m_H = 1.673D-27 ; mass of Hydrogen (kilograms)
m_gas = 44.0D * m_H ;mass of the CO_2 gas molecule
density = 3D3 ;3g/cm^3 - like avg. mafic silicates

; define array for altitude
numz = 36
z = dindgen(numz) * 2 ;km

;-----------------------------------------------------------------------------------------------

; read in initial temperature profile, and interpolate to z's under consideration
numlayers = 27
tp = gettp('/u/ngoldens/smithdat/PTavgTESclear.txt', numlayers) ;numlayers=27
pres = tp[0,*]
temp = tp[1,*]
scale_height = 10.8D ;1.08D4 ;10.8km
altitudes = (-1) * scale_height*alog(pres/pres[0])

temp_gas = spline(altitudes,temp,z)
pres_gas = spline(altitudes,pres,z)
ngas = (pres_gas*100.0D) / (kconst * temp_gas) ;P converted to Pascals
;------------------------define populations------------------------
;parameters of particle size distribution (see Hansen & Travis, 1974)
;define the particle size distribution at the bottom layer
r1 = 1D-7 ;r1 and r2 define the range of radii in the size dist
r2 = 1D-5
numr = 10.0D
logr = dblarr(numr)
logrw = dblarr(numr)
gaulge, alog(r1), alog(r2), logr, logrw, numr
radius = exp(logr)
logrw = logrw * radius

reff = 1.5D-6 ;effective radius of distribution, z=0
veff = 0.2D ;variance of distribution, z=0
constant = 1.0D ;adjusted later to depend on tau
n = constant * (radius ^ ((1.0D - (3.0D*veff[0]))/veff[0]))
    * exp(radius/(reff[0]*veff[0]))

;--------------------------define wavelength's under consideration--------------------------
// define wavelength's under consideration
lamda0 = 1D-7 ;4D-7 ;
lamdaMax = 1D-6
nlamda = 50
loglam = dblarr(nlamda)
loglamw = loglam
gaulge, alog(lamda0),alog(lamdaMax),loglam,loglamw,nlamda

lamda = exp(loglam)
loglamw = loglamw * lamda

lamda = [lamda, 9.3023D-6] ;reference wavelength is 1075cm^-1
ind = indgen(nlamda)
nlamda = nlamda + 1
;-------------------Mie stuff-----------------------------------------------
; where to get optical constants
filename = "/u/ngoldens/wolffscat/wolff06.txt" ; UNITS: microns, size=336
numlines = 336

; get optical constant data from file
m = dcomplexarr(3,numlines) ; complex index of refraction
m = getm(filename, numlines) ; get optical constants from file
wlarr = double(m[0,*]) * 1D-6
m_real = double(m[1,*])
m_imag = double(m[2,*])
; interpolate to wavelengths in use
m_real2 = interpol(m_real, wlarr, lamda)
m_imag2 = interpol(m_imag, wlarr, lamda)

Kabsarr = dbarr(nlamda,numr)
Kscarr = dbarr(nlamda,numr)
omegarr = dbarr(nlamda,numr)
cbararr = dbarr(nlamda,numr)

; loop over wavelength
for p = 0, nlamda-1 do begin
  m_lamda = dcomplex(m_real2[p], m_imag2[p])

  ; loop over radius
  for r = 0, numr-1 do begin
    nang=91
    Ksc = 0.0D & Kex = 0.0D & Kback = 0.0D & Gsca=0.0D & cbar = 0.0D
    S1 = dbarr(2*nang-1) & S2 = S1
    x = (2.D0 * !DPI * radius[r]) / lamda[p] ; x is dimensionless
    getbasicmie, radius[r], Ksc, Kex, lamda[p], m_lamda, cbar

    Kscarr[p,r] = Ksc
    omegarr[p,r] = Ksc/Kex
    Kabsarr[p,r] = Kex - Ksc
    cbararr[p,r] = cbar
  endfor
endfor ; wavelength loop
; define mixing ratios (q,z) number/volume
q = dblarr(numr, numz)

; - determines population distributions using the Conrath (1975)
; parameterization for the mixing ratio for each particle size (population) separately.
; define concentration q(z) as starting with each n(r) at the bottom
q[*,0] = n
H = 1D4              ; 10 km
K = 1D3              ; 1D3 -> x10^6 cm^2/s

gMars = 3.71D
 eta = 1.5D-5
lampath = 1.375D-5

d = 2.0D*radius       ; diameter
nu = dblarr(numr)
zm = z * 1000.0D     ; z needs to be in SI also

for i = 0, numr-1 do begin
  wzero = ((density*gMars*d[i]^2.0D)/(18.0D*eta))*(1.0D + 2.0D*(lampath/d[i]))
  nu[i] = H * wzero / K ; 0.030202518
  for up = 1, numz-1 do begin
    q[i,up] = q[i,0] * exp(nu[i]*(1.0D - exp(zm[up]/H)))
  endfor
endfor
; find constant c for particle size distribution before extending it up with altitude
wlref = nlamda-1 ; choose reference wavelength
reftau = 0.1768D ; 1.608D -- dusty
numlayers = numz - 1

tau_n = dblarr(numlayers) ; optical depth of each layer
tau_c = dblarr(numz) ; cumulative optical depth above each layer

for i=numlayers-1,0,-1 do begin ; top down
tau_n[i] = 0.0D
dz = zm[i+1] - zm[i]
for r=0, numr-1 do begin
  temptau = !DPI * radius[r]^2.0D * (kabsarr[wlref,r] + kscarr[wlref,r]) * ((q[r,i]+q[r,i+1])/2.0D) * dz
  if (r EQ 0 OR r EQ numr-1) then temptau = 0.5D * temptau
  tau_n[i] = tau_n[i] + temptau
endfor
tau_c[i] = tau_c[i+1] + tau_n[i]
endfor
tau_g = tau_c[0]

constant = reftau/tau_g

; adjust n(r) by new constant
n = constant * n
q = constant * q

; some defining
cosbar = 0.0D & beta_ab = 0.0D & beta_ex = 0.0D & two = 0.0D
miestruct = {allmie2, narr:n, reff:reff, veff:veff, g:cosbar, Qabs:beta_ab, Qex:beta_ex}
g = dblarr(nlamda,numlayers)
qabs = dblarr(nlamda,numlayers) & qex = dblarr(nlamda,numlayers)
omega = dblarr(nlamda,numlayers)
for p = 0, nlambda-1 do begin
    ; average Mie stuff over particle size distribution for each layer
    for d=0,numlayers-1 do begin
        ntemp = (q[*],d]+q[*],d+1]/2.0D
        miestruct = thesismie(radius, ntemp, miestruct, Kscarr[p,*],
                          Kabsarr[p,*],cbararr[p,*], logrw)
        g[p,d] = miestruct.g
        qabs[p,d] = miestruct.Qabs
        qex[p,d] = miestruct.Qex
    endfor
endfor

save, /variables, filename='results2/thesisK6clear.sav'
end

This next pre-processing subroutine calculates those values that need to be tabulated separately in the infrared, an IR wavelength array and Mie values.

pro popsir
    restore, 'results2/thesisK6clear.sav'
    ;------------------------------------------------------------------------
    ;// define wavelength's under consideration
    lamda0ir = 1D-6 ;4D-6 ;
    lamdaMaxir = 5D-5 ;5D-5 ;
    nlamdair = 140 ;40 ;
    loglamir = dubarr(nlamdair)
    loglamwir = loglamir
    gauleg, alog(lamda0ir),alog(lamdaMaxir),loglamir,loglamwir,nlamdair

    lamdaIR = exp(loglamir)
    loglamwir = loglamwir * lamdair
    lamdaIR = [lamdaIR, 9.3023D-6]
    nlamdair = nlamdair + 1
;--------------------Mie stuff---------------------
; interpolate to wavelengths in use
m_real2ir = interpol(m_real, wlarr, lamdaIR)
m_imag2ir = interpol(m_imag, wlarr, lamdaIR)

Kabsarrir = dblarr(nlamdaIR,numr)
Kscarrir = dblarr(nlamdaIR,numr)
omegarrir = dblarr(nlamdaIR,numr)
cbararrir = dblarr(nlamdaIR,numr)

;loop over wavelength
for p = 0, nlamdaIR-1 do begin
    print, "wavelength # ", p+1, " of ", nlamdaIR ; lamda array defined above
    m_lamda = dcomplex(m_real2ir[p], m_imag2ir[p])
endfor

;loop over radius
for r = 0, numr-1 do begin
    nang=91
    Ksc = 0.0D & Kex = 0.0D & Kback = 0.0D & Gsca=0.0D & cbar = 0.0D
    S1 = dblarr(2*nang-1) & S2 = S1
    x = (2.0D * !DPI * radius[r]) / lamdair[p] ; x is dimensionless
    getbasicmie, radius[r], Ksc, Kex, lamdair[p], m_lamda, cbar

    Kscarrir[p,r] = Ksc
    omegarrir[p,r] = Ksc/Kex
    Kabsarrir[p,r] = Kex - Ksc
    cbararrir[p,r] = cbar
endfor
endfor ;wavelength loop
cosbar = 0.0D & beta_ab = 0.0D & beta_ex = 0.0D & two = 0.0D
miestruct = {allmieir4, narr:n, reff:reff, veff:veff, g:cosbar, Qabs:beta_ab, Qex:beta_ex}
gIR = dblarr(nlamdaIR,numlayers)
qabsIR = dblarr(nlamdaIR,numlayers) & qexIR = dblarr(nlamdaIR,numlayers)

for p = 0, nlamdaIR-1 do begin
    ; average Mie stuff over particle size distribution for each layer
    for d=0,numlayers-1 do begin
        ntemp = (q[*],d] + q[*],d+1]/2.0D
        ntemp = ntemp / total(ntemp) ;normalized
        miestruct = thesismie(radius, ntemp, miestruct, Kscarrir[p,*],
                        Kabsarrir[p,*],, cbararrir[p,*], logrw)

        gIR[p,d] = miestruct.g
        qabsIR[p,d] = miestruct.Qabs
        qexIR[p,d] = miestruct.Qex
    endfor
endfor

save, /variables, filename='results2/thesisIRK6clear.sav'
end
B.2.2 Solar-band radiative transfer solution (after Toon et al., 1989)

This code restores the pre-processing results, calculates optical depths, and finds mean intensity and net flux at all layers in the atmosphere.

pro solarrad
  restore, 'results2/thesisK6clear.sav'

; stuff to define
Tsun = 5800.0D ; SI units
Rsun = 6.9598D8
Dsun = 2.28D11
muzero = cos(rad(34.0D)) ; solar incidence angle

numlayers = numz-1
; declare for solar part
Cplus = dblarr(numlayers, 2)
Cminus = dblarr(numlayers, 2)
e = dblarr(4, numlayers)
l = 2 * numlayers
A = dblarr(l) & B=A & D=A & Ebig=A & u2=A & Y=A
direct = dblarr(nlamda, numlayers)
Fnet = dblarr(nlamda, numlayers)
J = dblarr(nlamda, numlayers)

; calculate absorption from solar radiation-----------------------------
for wl=0, nlamda-2 do begin
  ; calculate optical depths
  for i=numlayers-1,0,-1 do begin ; top down
    dz = zm[i+1] - zm[i]
    tau_n[i] = 0.0D
    for r = 0, numr-1 do begin
      temptau = !DPI * radius[r]^2.0D * (kabsarr[wl, r]+kscarr[wl,r])
      * ((q[r,i]+q[r,i+1])/2.0D) * dz
      if (r EQ 0 OR r EQ numr-1) then temptau = 0.5D * temptau
      tau_n[i] = tau_n[i] + temptau
    endfor
  ; delta scaling:
    omegazero = (qex[wl,i] - qabs[wl,i])/qex[wl,i]
    tau_n[i] = tau_n[i] * (1.0D - (omegazero * g[wl,i]^2.0D))
  ; add up cumulative tau_c
    tau_c[i] = tau_c[i+1] + tau_n[i]
  endfor
\( \tau_g = \tau_c[0] \); total optical depth at the ground
\( F_{\text{top}} = \text{planck}(\lambda_{wl}, T_{\text{Sun}}) \times (R_{\text{Sun}}^2) / (D_{\text{Sun}}^2) \)
reflectivity = 0.15D
\( S_{\text{sfc}} = \text{reflectivity} \times m_{\text{zero}} \times \exp(-\tau_c[1]/m_{\text{zero}}) \times \! \text{DPI} \times F_{\text{top}} \)
print, \( \tau_g \)

; fill up arrays:
for \( i = \text{numlayers} - 1, 0, -1 \) do begin

\( \omega_{\text{zero}} = (q_{\text{ex}}[wl,i] - q_{\text{abs}}[wl,i])/q_{\text{ex}}[wl,i] \)
\( \text{asymm} = g[wl,i] \)

; delta scalings:
\( \omega_{\text{zero}} = \omega_{\text{zero}} \times ((1.0D - \text{asymm}^2.0D) / (1.0D - \omega_{\text{zero}} \times \text{asymm}^2.0D)) \)
if \( \omega_{\text{zero}} \) EQ 1 then \( \omega_{\text{zero}} = 0.9999999999999D \)
\( \text{asymm} = \text{asymm} / (1.0D + \text{asymm}) \)

\( \gamma_1 = (7.0D - \omega_{\text{zero}} \times (4.0D + (3.0D \times \text{asymm}))) / 4.0D \)
\( \gamma_2 = -1.0D \times (1.0D - \omega_{\text{zero}} \times (4.0D - (3.0D \times \text{asymm}))) / 4.0D \)
\( \text{templamb} = (\gamma_1^2.0D - \gamma_2^2.0D)^{0.5D} \)
\( \text{biggam} = (\gamma_1 - \text{templamb}) / \gamma_2 \)
\( \mu_1 = (1.0D - \omega_{\text{zero}})/(\gamma_1 - \gamma_2) \)
\( \gamma_3 = (2.0D - (3.0D \times \text{asymm} \times m_{\text{zero}}))/4.0D \)
\( \gamma_4 = 1.0D - \gamma_3 \)
\( \text{Ctemp} = (((\gamma_1 - (1.0D/m_{\text{zero}})) \times \gamma_3 + (\gamma_4 \times \gamma_2)) / ((\text{templamb}^2.0D) - (1.0D/(m_{\text{zero}}^2.0D))) \)
\( \text{Cplus}[i,0] = \omega_{\text{zero}} \times \! \text{DPI} \times F_{\text{top}} \times \exp((-\tau_c[i+1]/m_{\text{zero}})) \times \text{Ctemp} \)
\( \text{Cminus}[i,0] = (\omega_{\text{zero}} \times \! \text{DPI} \times F_{\text{top}} \times \exp((-\tau_c[i+1]/m_{\text{zero}}))) \times \gamma_1 \times (\gamma_4 + (\gamma_2 \times \gamma_3)) / ((\text{templamb}^2.0D) - (1.0D/(m_{\text{zero}}^2.0D))) \)
\( \text{Cplus}[i,1] = \omega_{\text{zero}} \times \! \text{DPI} \times F_{\text{top}} \times \exp((-\tau_c[i+1]-\tau_n[i]/m_{\text{zero}})) \times \text{Ctemp} \)
\( \text{Cminus}[i,1] = (\omega_{\text{zero}} \times \! \text{DPI} \times F_{\text{top}} \times \exp((-\tau_c[i+1]-\tau_n[i]/m_{\text{zero}}))) \times \gamma_1 \times (\gamma_4 + (\gamma_2 \times \gamma_3)) / ((\text{templamb}^2.0D) - (1.0D/(m_{\text{zero}}^2.0D))) \)
\( e[0,i] = 1.0D + \text{biggam} \times \exp(-\text{templamb} \times \tau_n[i]) \)
\( e[1,i] = 1.0D - \text{biggam} \times \exp(-\text{templamb} \times \tau_n[i]) \)
\( e[2,i] = \text{biggam} \times \exp(-\text{templamb} \times \tau_n[i]) \)
\( e[3,i] = \text{biggam} \times \exp(-\text{templamb} \times \tau_n[i]) \)
endfor
;fill tridiagonal matrix arrays
n=0
i = numlayers - 1 - n
A[0] = 0.0D
B[0] = e[0,i]
D[0] = -e[1,i]
Ebig[0] = 0.0D - Cminus[i,0]

for l = 1, (2*numlayers)-2, 2 do begin
; odd
A[l+1] = (e[1,i]*e[2,i]) - (e[3,i]*e[0,i])
B[l+1] = (e[0,i]*e[0,i-1]) - (e[2,i]*e[2,i-1])
D[l+1] = (e[2,i]*e[3,i-1]) - (e[0,i]*e[1,i-1])
Ebig[l+1] = e[2,i]*(Cplus[i-1,0] - Cplus[i,1]) + e[0,i]*(Cminus[i,1] - Cminus[i-1,0])

; even
A[l] = (e[1,i-1]*e[0,i]) - (e[2,i]*e[3,i-1])
B[l] = (e[1,i]*e[1,i-1]) - (e[3,i]*e[3,i-1])
D[l] = (e[0,i-1]*e[3,i-1]) - (e[1,i-1]*e[2,i-1])
Ebig[l] = e[1,i-1]*(Cplus[i-1,0] - Cplus[i,1]) - e[3,i-1]*(Cminus[i-1,0] - Cminus[i,1])

n=n+1
i = numlayers - 1 - n
endfor

i=0
l = (2.0D*numlayers) - 1.0D
A[l] = e[0,i] - (reflectivity * e[2,i])
B[l] = e[1,i] - (reflectivity * e[3,i])
D[l] = 0.0D
Ebig[l] = Ssfc - Cplus[i,1] + (reflectivity * Cminus[i,1])

; prepare to solve:
; A & D arrays are n-1 big
ind = where(A NE 0)
A1 = A[ind]
ind = where(D NE 0)
D1 = D[ind]
; decompose arrays
B1 = B
LA_TRIDC, A1, B1, D1, u2, index ; need to define u2 and index?
; get solution
Y = LA_TRISOL(A1, B1, D1, u2, index, Ebig)

for n = 0, numlayers-1 do begin
  nn = n + 1
  i = numlayers - 1 - n
  direct[wl,n] = muzero * !DPI * Ftop * exp((-tau_c[i+1]-tau_n[i])/muzero)
  Fnet[wl,n] = Y[2*nn-2]*(e[0,i]-e[2,i]) + Y[2*nn-1]*(e[1,i]-e[3,i])
                 + Cplus[i,1] - Cminus[i,1] - direct[wl,n]
  J[wl,n] = ((Y[2*nn-2]*(e[0,i]+e[2,i])) + (Y[2*nn-1]*(e[1,i]+e[3,i])))
           + Cplus[i,1] + Cminus[i,1]) * (1.0D/(4.0D*!DPI*mu1))
  J[wl,n] = J[wl,n] + direct[wl,n]/(4.0D*!DPI*muzero)
endfor

endfor ;loop over wavelength

save, /variables, filename='results2/solarK6clear.sav'
end
B.3 Radiative/Conductive Equilibrium

This code calculates the mean intensities in the infrared, finds the equilibrium dust temperature at a particles size at a time, and adjusts the infrared mean intensities based on the new temperatures until they converge.

```plaintext
pro radeql
    restore, 'results2/thesisIRK6clear.sav'

    mu = cos(rad(34.0D))  ;solar incidence angle (for clear: 34, for dusty: 43)

    temp_grd = 291.9D  ;260.82D: dusty
    tempold = dblarr(numr,numz)
    tempnew = tempold
    for d=0,numz-1 do tempold[*,d] = temp_gas[d]

    zm = z * 1000.0D  ;in units of meters
    tau_c = dblarr(numz)
    tau_n = dblarr(numlayers)

    ;------------------------------------------------------------------------------------------------------------------------
    restore, 'results2/solarK6clear.sav'
    Jsolar = J
    for r=0,numr-1 do begin
        bigx = dblarr(numlayers)  ;also do for each layer, j is at the bottom of each layer
        for d = 0, numlayers-1 do begin
            for wl=0, nlamda-2 do begin
                bigx[d] = bigx[d] + kabsarr[wl,r]*Jsolar[wl,d]*loglamw[wl]
            endfor
            bigx[d] = bigx[d] * !DPI * 4.0D
        endfor
        bigx = reverse(bigx)
    ;------------------------------------------------------------------------------------------------------------------------
    tau_g = dblarr(nlamdair-1)
    print, 'radius = ', radius[r]
    bigloop=0
    while bigloop EQ 0 do begin
        Jir = dblarr(nlamdair,numz)
        for wl=0,nlamdair-2 do begin
            ;calculate intensity field in the infrared
```
; calculate optical depths
tau_c[numz-1]=0.0D
for i=numlayers-1,0,-1 do begin ; top down
dz = zm[i+1] - zm[i]
tau_n[i] = 0.0D
tau_c[i] = 0.0D
for r1=0, numr-1 do begin
  temptau = DPI * radius[r1]^2.0D * (kabsarrIR[wl,r1] + kscarrIR[wl,r1]) * ((q[r1,i]+q[r1,i+1])/2.0D) * dz
  if (r1 EQ 0 OR r1 EQ numr-1) then temptau = 0.5D * temptau
  tau_n[i] = tau_n[i] + temptau
endfor
tau_c[i] = tau_c[i+1] + tau_n[i]
endfor
tau_g[wl] = tau_c[0]
Bgrd = planck(lamdair[wl], temp_grd)
; loop over optical depth to get avg. intensities
Jirtemp=0.0D
for i=0,numz-2 do begin
  for j=0, i-1 do begin
    Bavg = (planck(lamdair[wl], temptemp[j]) + planck(lamdair[wl], temptemp[j+1]))/2.0D
    arg1 = EXPINT(2, tau_c[j+1]-tau_c[i])
    arg2 = EXPINT(2, tau_c[j]-tau_c[i])
    Jirtemp = Jirtemp + (Bavg * (arg1 - arg2))
  endfor
  Jir[wl,i] = 0.5D * Jirtemp
endfor
Jirtemp = 0.0D
for j=i,numlayers-1 do begin
  Bavg = (planck(lamdair[wl], temptemp[j]) + planck(lamdair[wl], temptemp[j+1]))/2.0D
  arg1 = EXPINT(2, tau_c[i]-tau_c[j])
  arg2 = EXPINT(2, tau_c[i]-tau_c[j+1])
  Jirtemp = Jirtemp + (Bavg * (arg1 - arg2))
endfor
Jir[wl,i] = Jir[wl,i] + 0.5D*Jirtemp
Jir[wl,i] = Jir[wl,i] + (0.5D * Bgrd * EXPINT(2,tau_c[0]-tau_c[i]))
endfor ; end loop over wavelength
IRabs = dblarr(numz)
for d = 0, numz-1 do begin ;for each level, integrate over wavelength
  for w = 0, nlambda-2 do begin
    IRabs[d] = IRabs[d] + kabsarrIR[w,r] * Jir[w,l,d] * loglamwir[w]
  endfor
endfor
IRabs[d] = IRabs[d] * 4.0D * !DPI
endfor

; check on radiative equilibrium  -----------------------------------------------
for d=0, numz-2 do begin
  v = ((8.D0 * kconst * temp_gas[d])/(!DPI * m_gas))^(0.5D)
  Utot = inteng(temp_gas[d])
  const = bigx[d] + IRabs[d] + (ngas[d] * v * Utot)

  ; limiting temperatures:
  t1 = 50.0D
  t2 = 1000.0D

  finish = 0
  while finish EQ 0 do begin
    thalf = (t1 + t2)/2.0D
    IRout = 0.0D
    for w = 0, nlambda-2 do begin
      IRout = IRout + (kabsarrIR[w,r]*planck(lamdair[w],thalf)*loglamwir[w])
    endfor
endfor
  Uhlf = inteng(thalf);(5.0D/2.0D)*kconst*thalf;
  value = (4.0D * !DPI * IRout) + (ngas[d] * v * Uhlf)

  if (value GT const) then t2 = thalf else if (value LT const) then t1 = thalf
  if abs(t1 - t2) LT 0.1 then finish = 1
endwhile

  tempnew[r,d] = thalf
endfor

;check variance of temps
if abs(tempnew[r,numz-1] - tempold[r,numz-1]) gt 0.1D
  then tempold[r,*]=tempnew[r,*] else bigloop=1
endwhile
endfor ;loop over radii
save, /variables, filename='results2/theendK6clear.sav'
end
This code, called by ‘radeql’ calculates part of the conductive heat transfer term.

```
function inteng, temp
; calculate internal energy for CO2 at given temperature

h = 6.6260755D-34  ; Planck constant (Joule * sec)
kconst = 1.380658D-23  ; Boltzmann constant (Joules/Kelvin)
c = 2.99792458D8    ; speed of light (m/s)

nu = [1388.3, 667.3, 2349.3]    ; wavenumbers of absorptions
nu = (1.0D/nu)/100.0D             ; wavelength in m
nu = c / nu                     ; frequency in Hz

Utr = (3.0D/2.0D)*kconst*temp    ; translational
Urot = kconst*temp               ; rotational
Uvib = 0.0D                      ; vibrational

for j = 0, 2 do begin
    theta = h * nu[j] / kconst
    Uvib = Uvib + (theta/(exp(theta/temp)-1.0D))
endfor
Uvib = Uvib * kconst
U = Utr + Urot + Uvib

return, U
end
```